A Finite Volume Kinetic (FVK) framework for unsteady mixed flows in non uniform closed water pipes.

Mehmet Ersoy [1], Christian Bourdarias [2] and Stéphane Gerbi [3]

Euskadi - Kyushu 2011

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Outline of the talk

1. Unsteady mixed flows : PFS equations (Pressurized and Free Surface)
    - Definitions and examples
    - The PFS model
2. A FVK Framework
    - Kinetic Formulation and numerical scheme
    - Numerical results
3. Conclusion and perspectives

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Unsteady mixed flows in closed water pipes?

- Free surface area (SL)
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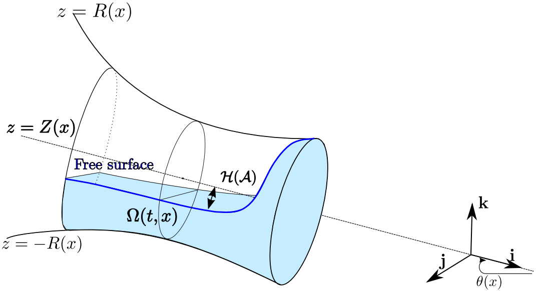

Unsteady mixed flows in closed water pipes?

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- Pressurized area (CH)
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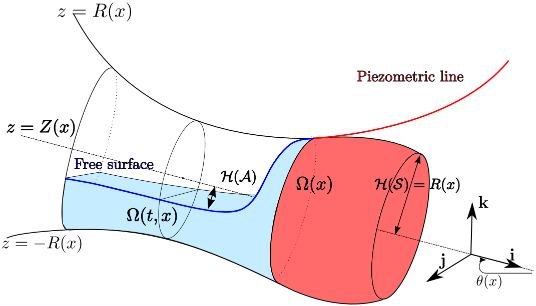
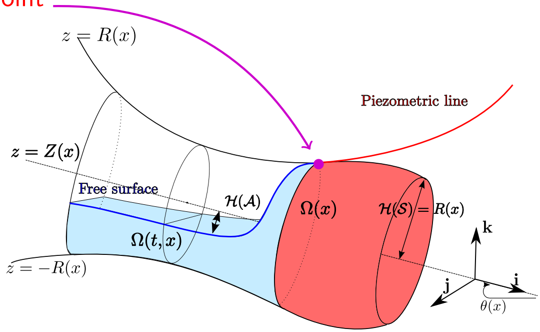
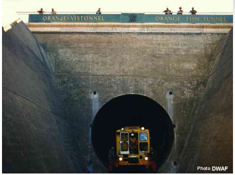


Unsteady mixed flows in closed water pipes ?

- Free surface area (SL)
  sections are not completely filled and the flow is incompressible...
- Pressurized area (CH)
  sections are non completely filled and the flow is compressible...
- Transition point



Examples of pipes



Orange-Fish tunnel

Sewers . . . in Paris

Forced pipe

problems . . . at Minnesota
`http://www.sewerhistory.org/grfx/misc/disaster.htm`

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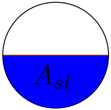
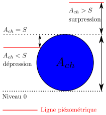
The PFS model

$$\begin{cases} \partial_t(A) + \partial_x(Q) & = 0 \\ \partial_t(Q) + \partial_x\left(\dfrac{Q^2}{A} + p(x,A,E)\right) & = -g\,A\,\dfrac{d}{dx}Z(x) \\ & +Pr(x,A,E) \\ & -G(x,A,E) \\ & -g\,K(x,\mathbf{S})\,\dfrac{Q|Q|}{A} \end{cases}$$

with $A = \begin{cases} A_{sl} & \text{if} \quad E = 0 \text{ (FS state)}, \\ A_{ch} & \text{if} \quad E = 1 \text{ (P state)}, \end{cases}$ $\mathbf{S} = \begin{cases} A_{sl} & \text{if} \quad E = 0, \\ S = |\Omega(x)| & \text{if} \quad E = 1, \end{cases}$





and ...

C. Bourdarias, M. Ersoy and S. Gerbi
A model for unsteady mixed flows in non uniform closed water pipes and a well-balanced finite volume scheme.
Int. J. On Finite Volumes, 6(2) :1–47, 2009.

Pressure and source terms

$$
\begin{aligned}
p &= c^2(A - \mathbf{S}) + gI_1(x, \mathbf{S})\cos\theta && : \text{mixed pressure law} \\
Pr &= c^2\left(\frac{A_{ch}}{S} - 1\right)\frac{d\,S}{dx} + gI_2(\mathbf{S})\cos\theta && : \text{pressure source term} \\
G &= gA\overline{z}\frac{d}{dx}\cos\theta && : \text{curvature source term} \\
K &= \frac{1}{K_s^2 R_h(\mathbf{S})^{4/3}} && : \text{Manning-Strickler law}
\end{aligned}
$$

The PFS model

Mathematical properties

- The **PFS** system is strictly hyperbolic for $A(t,x) > 0$.
- For regular solutions, the mean speed $u = Q/A$ verifies

$$\partial_t u + \partial_x \left(\frac{u^2}{2} + c^2 \ln(A/S) + g\,\mathcal{H}(S)\cos\theta + g\,Z \right) = -g\,K(x,\mathbf{S})\,u\,|u|$$

  and for $u = 0$, we have :

$$c^2 \ln(A/\mathbf{S}) + g\,\mathcal{H}(\mathbf{S})\cos\theta + g\,Z = cte$$

  where $\mathcal{H}(\mathbf{S})$ is the physical water height.
- There exists a mathematical entropy

$$E(A,Q,S) = \frac{Q^2}{2A} + c^2 A \ln(A/\mathbf{S}) + c^2 S + g\overline{z}(x,\mathbf{S})\cos\theta + gAZ$$

  which satisfies

$$\partial_t E + \partial_x \left(E\,u + p(x,A,E)\,u \right) = -g\,A\,K(x,\mathbf{S})\,u^2\,|u| \leqslant 0$$

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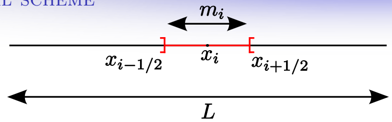
Finite Volume (VF) numerical scheme of order 1

Cell-centered numerical scheme



PFS equations under vectorial form :

$$\partial_t \mathbf{U}(t,x) + \partial_x F(x,\mathbf{U}) = \mathcal{S}(t,x)$$

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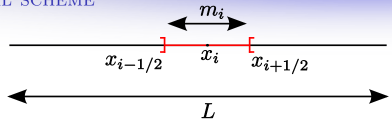


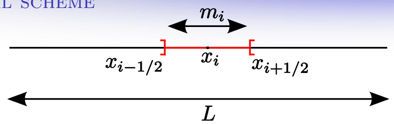
PFS equations under vectorial form :

$$\partial_t \mathbf{U}(t,x) + \partial_x F(x,\mathbf{U}) = \mathcal{S}(t,x)$$

with $\mathbf{U}_i^n \overset{\text{cte per mesh}}{\approx} \dfrac{1}{\Delta x} \displaystyle\int_{m_i} \mathbf{U}(t_n,x)\,dx$ and $\mathcal{S}(t,x)$ constant per mesh,

FINITE VOLUME (VF) NUMERICAL SCHEME OF ORDER 1

CELL-CENTERED NUMERICAL SCHEME



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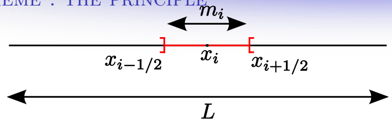
Cell-centered numerical scheme :

$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \frac{\Delta t^n}{\Delta x}\left(\mathcal{F}_{i+1/2}(\mathbf{U}_i,\mathbf{U}_{i-1}) - \mathcal{F}_{i-1/2}(\mathbf{U}_{i-1},\mathbf{U}_i)\right) + \Delta t^n \mathcal{S}(\mathbf{U}_i^n)$$

where

$$\Delta t^n \mathcal{S}_i^n \approx \int_{t_n}^{t_{n+1}} \int_{m_i} \mathcal{S}(t,x)\,dx\,dt$$

Upwinded numerical scheme : the principle



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Upwinded numerical scheme :

$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \frac{\Delta t^n}{\Delta x}\left(\widetilde{\mathcal{F}}_{i+1/2}(\mathbf{U}_i, \mathbf{U}_{i-1}, \mathcal{S}_{i,i+1}) - \widetilde{\mathcal{F}}_{i-1/2}(\mathbf{U}_{i-1}, \mathbf{U}_i, \mathcal{S}_{i-1,i})\right)$$

Choice of the numerical fluxes

Our goal : define $\mathcal{F}_{i+1/2}$ in order to preserve continuous properties of the PFS-model

Positivity of A ,

conservativity of A, discrete equilibrium, discrete entropy inequality

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Our choice

VFRoe solver[BEGVF] Kinetic solver[BEG10]

C. Bourdarias, M. Ersoy and S. Gerbi.
A model for unsteady mixed flows in non uniform closed water pipes and a well-balanced finite volume scheme.
International Journal On Finite Volumes , Vol 6(2) 1–47, 2009.

C. Bourdarias, M. Ersoy and S. Gerbi.
A kinetic scheme for transient mixed flows in non uniform closed pipes : a global manner to upwind all the source terms.
J. Sci. Comp.,pp 1-16, 10.1007/s10915-010-9456-0, 2011.

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We introduce

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then we define the Gibbs equilibrium by

$$\mathcal{M}(t,x,\xi) = \frac{A(t,x)}{b(t,x)} \chi\left(\frac{\xi - u(t,x)}{b(t,x)}\right)$$

with

$$b(t,x) = \sqrt{\frac{p(t,x)}{A(t,x)}}$$

Since

$$\chi(\omega) = \chi(-\omega) \geq 0 \,, \int_{\mathbb{R}} \chi(\omega) d\omega = 1, \int_{\mathbb{R}} \omega^2 \chi(\omega) d\omega = 1 \,,$$

and

$$\mathcal{M}(t,x,\xi) = \frac{A(t,x)}{b(t,x)} \,\chi\left(\frac{\xi - u(t,x)}{b(t,x)}\right)$$

then

$$\begin{aligned} A &= \int_{\mathbb{R}} \mathcal{M}(t,x,\xi)\, d\xi \\ Q &= \int_{\mathbb{R}} \xi \mathcal{M}(t,x,\xi)\, d\xi \\ \frac{Q^2}{A} + \underbrace{A\, b^2}_{p} &= \int_{\mathbb{R}} \xi^2 \mathcal{M}(t,x,\xi)\, d\xi \end{aligned}$$

Principle [P02]

The kinetic formulation

(A, Q) is solution of the PFS system if and only if $\mathcal{M}$ satisfy the transport equation :

$$\partial_t \mathcal{M} + \xi \cdot \partial_x \mathcal{M} - g\Phi\, \partial_\xi \mathcal{M} = \mathcal{K}(t, x, \xi)$$

where $\mathcal{K}(t, x, \xi)$ is a collision kernel satisfying a.e. (t, x)

$$\int_{\mathbb{R}} \mathcal{K}\, d\xi = 0\ ,\ \int_{\mathbb{R}} \xi\, \mathcal{K} d\,\xi = 0$$

and Φ are the source terms.

B. Perthame.
Kinetic formulation of conservation laws.
Oxford University Press.
Oxford Lecture Series in Mathematics and its Applications, Vol 21, 2002.

Principe

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and Φ are the source terms.

General form of the source terms :

$$\Phi = \overbrace{\frac{d}{dx} Z}^{\text{conservative}} + \overbrace{\mathbf{B} \cdot \frac{d}{dx} \mathbf{W}}^{\text{non conservative}} + \overbrace{K \frac{Q|Q|}{A^2}}^{\text{friction}}$$

with $\mathbf{W} = (Z, S, \cos\theta)$

Discretization of source terms

- Recalling that A, Q and $Z, S, \cos\theta$ constant per mesh
- forgetting the friction : « splitting »...

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Then $\forall (t,x) \in [t_n, t_{n+1}[\times \overset{\circ}{m}_i$

$$\Phi(t,x) = 0$$

since

$$\Phi = \frac{d}{dx} Z + \mathbf{B} \cdot \frac{d}{dx} \mathbf{W}$$

Simplification of the transport equation

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$\Longrightarrow$

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$\Longrightarrow$

$$\left\{ \begin{array}{rcl} \partial_t f + \xi \cdot \partial_x f & = & 0 \\ f(t_n, x, \xi) & = & \mathcal{M}(t_n, x, \xi) \overset{def}{:=} \dfrac{A(t_n, x, \xi)}{b(t_n, x, \xi)} \chi \left(\dfrac{\xi - u(t_n, x, \xi)}{b(t_n, x, \xi)} \right) \end{array} \right.$$

by neglecting the collision kernel

Discretization of source terms

On $[t_n, t_{n+1}[\times m_i$, we have :

$$\left\{\begin{array}{lcl} \partial_t f + \xi \cdot \partial_x f & = & 0 \\ f(t_n, x, \xi) & = & \mathcal{M}_i^n(\xi) \end{array}\right.$$

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i.e.

$$f_i^{n+1}(\xi) = \mathcal{M}_i^n(\xi) + \xi \frac{\Delta t^n}{\Delta x} \left(\mathcal{M}^-_{i+\frac{1}{2}}(\xi) - \mathcal{M}^+_{i-\frac{1}{2}}(\xi) \right)$$

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where

$$\mathbf{U}_i^{n+1} = \begin{pmatrix} A_i^{n+1} \\ Q_i^{n+1} \end{pmatrix} \overset{def}{:=} \int_{\mathbb{R}} \begin{pmatrix} 1 \\ \xi \end{pmatrix} f_i^{n+1}(\xi)\, d\xi$$

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or

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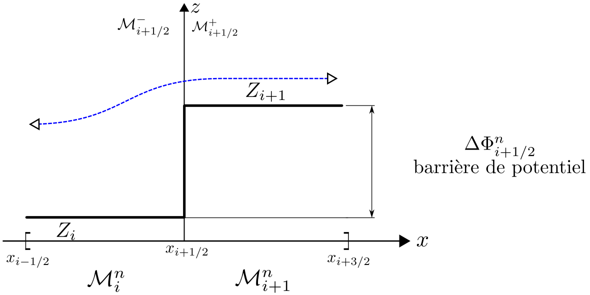
with

$$\widetilde{\mathcal{F}}_{i\pm\frac{1}{2}}^\pm = \int_{\mathbb{R}} \xi \left(\begin{array}{c} 1 \\ \xi \end{array} \right) \mathcal{M}_{i\pm\frac{1}{2}}^\pm(\xi)\, d\xi.$$

The microscopic fluxes

Interpretation : potential bareer

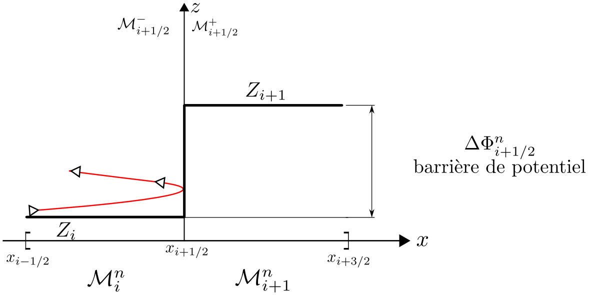
$$\mathcal{M}^-_{i+1/2}(\xi) = \overbrace{\mathbb{1}_{\{\xi>0\}}\mathcal{M}^n_i(\xi)}^{\text{positive transmission}}$$
$$+ \underbrace{\mathbb{1}_{\{\xi<0,\,\xi^2-2g\Delta\Phi^n_{i+1/2}>0\}}\mathcal{M}^n_{i+1}\left(-\sqrt{\xi^2-2g\Delta\Phi^n_{i+1/2}}\right)}_{\text{negative transmission}}$$



The microscopic fluxes

Interpretation : potential bareer

$$\mathcal{M}^-_{i+1/2}(\xi) = \overbrace{\mathbb{1}_{\{\xi>0\}}\mathcal{M}^n_i(\xi)}^{\text{positive transmission}} + \overbrace{\mathbb{1}_{\{\xi<0,\, \xi^2-2g\Delta\Phi^n_{i+1/2}<0\}}\mathcal{M}^n_i(-\xi)}^{\text{reflection}}$$
$$+ \underbrace{\mathbb{1}_{\{\xi<0,\, \xi^2-2g\Delta\Phi^n_{i+1/2}>0\}}\mathcal{M}^n_{i+1}\left(-\sqrt{\xi^2-2g\Delta\Phi^n_{i+1/2}}\right)}_{\text{negative transmission}}$$



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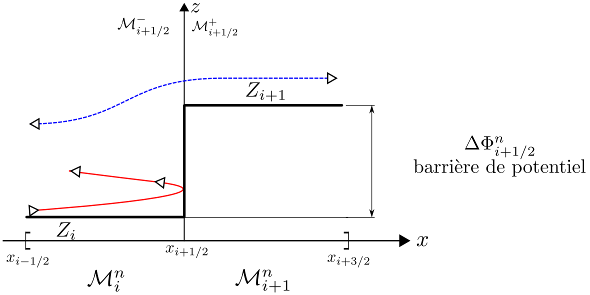

$\Delta\Phi^n_{i+1/2}$ may be interpreted as a time-dependant slope !

The microscopic fluxes

Interpretation : Dynamic slope $\Longrightarrow$ Upwinding of the friction

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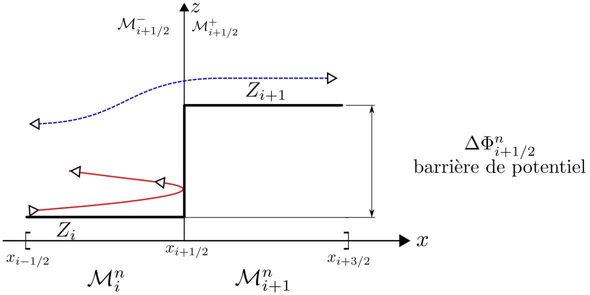

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. . . we reintegrate the friction . . .

Upwinding of the source terms

- conservative $\partial_x \mathbf{W}$:

$$\mathbf{W}_{i+1} - \mathbf{W}_i$$

- non-conservative $\mathbf{B}\partial_x \mathbf{W}$:

$$\overline{\mathbf{B}}(\mathbf{W}_{i+1} - \mathbf{W}_i)$$

where

$$\overline{\mathbf{B}} = \int_0^1 \mathbf{B}(s, \phi(s, \mathbf{W}_i, \mathbf{W}_{i+1}))\, ds$$

for the « straight lines paths », i.e.

$$\phi(s, \mathbf{W}_i, \mathbf{W}_{i+1}) = s\mathbf{W}_{i+1} + (1-s)\mathbf{W}_i,\ s \in [0,1]$$

G. Dal Maso, P. G. Lefloch and F. Murat.
Definition and weak stability of nonconservative products.
J. Math. Pures Appl. , Vol 74(6) 483–548, 1995.

Numerical properties

With [ABP00]

$$\chi(\omega) = \frac{1}{2\sqrt{3}} \mathbb{1}_{[-\sqrt{3},\sqrt{3}]}(\omega)$$

we have :

- Positivity of A (under a CFL condition),
- Conservativity of A,
- Natural treatment of drying and flooding area.

for example

E. Audusse and M-0. Bristeau and B. Perthame.
Kinetic schemes for Saint-Venant equations with source terms on unstructured grids.
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$\longrightarrow$ but easy computation of the numerical fluxes

E. Audusse and M-0. Bristeau and B. Perthame.
Kinetic schemes for Saint-Venant equations with source terms on unstructured grids.
INRIA Report RR3989, 2000.

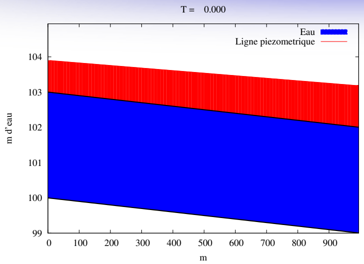
OUTLINE

1. UNSTEADY MIXED FLOWS : PFS EQUATIONS (PRESSURIZED AND FREE SURFACE)
   - Definitions and examples
   - The PFS model

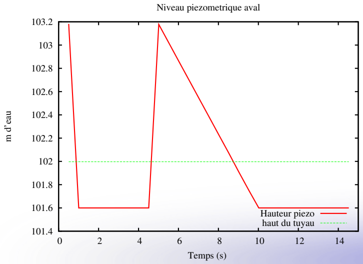
2. A FVK FRAMEWORK
   - Kinetic Formulation and numerical scheme
   - Numerical results

3. CONCLUSION AND PERSPECTIVES

Qualitative analysis of convergence



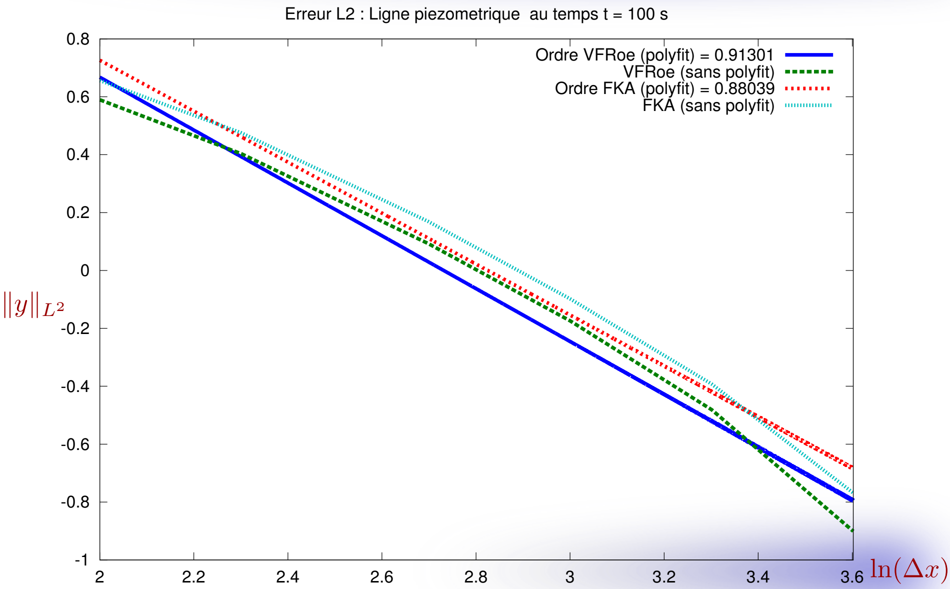
- upstream piezometric head $104\ m$



- downstream piezometric head :

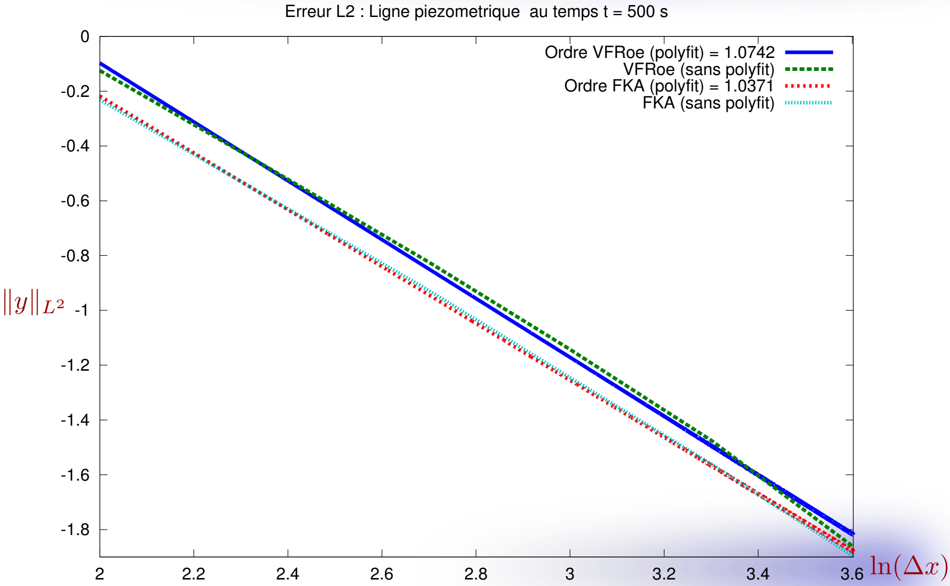
Convergence

During unsteady flows $t = 100\ s$



Convergence

Stationary $t = 500\ s$



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Conclusion

- ✅ Easy implementation even if source terms are complex.
- ✅ Very good agreement for uniform pipe.

Conclusion and perspectives

✅ Easy implementation even if source terms are complex.

✅ Very good agreement for uniform pipe.

To do :

- Upwinding scheme $\Longrightarrow$ *a priori* preservation of steady states (in progress).
- Discrete entropy inequalities ?

Thank you

for your

attention

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