Air entrainment in transient flows in closed water pipes

A two-layer approach.

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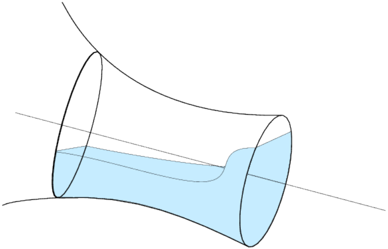
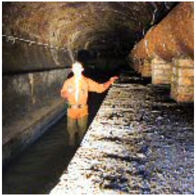
The air entrainment appears in the transient flow in closed pipes not completely filled: the liquid flow (as well as the air flow) is free surface.

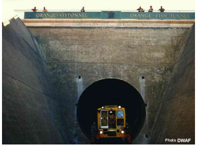


Some closed pipes

a forced pipe



a sewer in Paris



The Orange-Fish Tunnel (in Canada)

- the homogeneous model: a single fluid is considered where sound speed depends on the fraction of air
  M. H. Chaudhry *et al.* 1990 and Wylie an Streeter 1993
- the drift-flux model: the velocity fields are expressed in terms of the mixture center-of-mass velocity and the drift velocity of the vapor phase
  Ishii *et al.* 2003, Faille and Heintze 1999.
- The two-fluid model : a compressible and incompressible model are coupled via the interface. PDE of 6 equations
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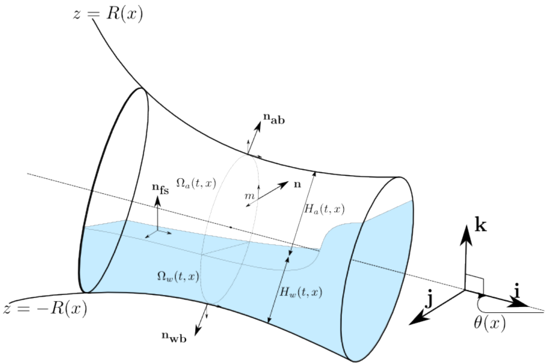

Figure: Geometric characteristics of the domain

We have then the first natural coupling:

$$H_w(t,x) + H_a(t,x) = 2R(x)\,.$$

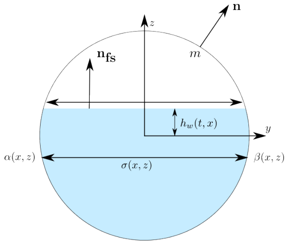

Figure: Cross-section of the domain

Incompressible Euler's equations

$$
\begin{array}{rcll}
\mathrm{div}(\mathbf{U_w}) & = & 0, & \text{on } \mathbb{R} \times \Omega_{t,w} \\
\partial_t(\mathbf{U_w}) + \mathrm{div}(\mathbf{U_w} \otimes \mathbf{U_w}) + \nabla P_w & = & \mathbf{F}, & \text{on } \mathbb{R} \times \Omega_{t,w}
\end{array}
$$

where $\mathbf{U_w}(t,x,y,z) = (U_w, V_w, W_w)$ the velocity , $P_w(t,x,y,z)$ the pressure , $\mathbf{F}$ the gravity strength.

- $A(t,x) = \displaystyle\int_{\Omega_w} dydz$: the wetted area,
- u the mean value of the speed
  $u(t,x) = \dfrac{1}{A(t,x)} \displaystyle\int_{\Omega_w} U_w(t,x,y,z)\, dydz,$
- $Q(t,x) = A(t,x)u(t,x)$ the discharge.

1. Averaged values in the Euler's Equations
2. Equality of the pressure of air and water at the free surface $P_a = P_w$ on fs
3. No slip condition on the bottom
4. Averaged nonlinearity $\simeq$ Nonlinearity of the averaged

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Fluid layer model

$$\left\{\begin{array}{lcl}
\partial_t A + \partial_x Q & = & 0 \\
\partial_t Q + \partial_x \left(\dfrac{Q^2}{A} + A P_a(\overline{\rho})/\rho_0 + g I_1(x,A)\cos\theta \right) & = & -gA\partial_x Z \\
 & & + g I_2(x,A)\cos\theta \\
 & & + P_a(\overline{\rho})/\rho_0\, \partial_x A
\end{array}\right.$$

g : the gravity constant, θ: the angle

$$I_1(x,A) = \int_{-R}^{h_w} (h_w - z)\sigma(x,z)\,dz \text{ the hydrostatic pressure}$$

$$I_2(x,A) = \int_{-R}^{h_w} (h_w - z)\partial_x\sigma(x,z)\,dz \text{ the pressure source term}$$

Compressible Euler's equations

$$\begin{array}{rcll} \partial_t \rho_a + \mathrm{div}(\rho_a \mathbf{U_a}) & = & 0, & \text{on } \mathbb{R} \times \Omega_{t,a} \\ \partial_t(\rho_a \mathbf{U_a}) + \mathrm{div}(\rho_a \mathbf{U_a} \otimes \mathbf{U_a}) + \nabla P_a & = & 0, & \text{on } \mathbb{R} \times \Omega_{t,a} \end{array}$$

where $\mathbf{U_a}(t, x, y, z) = (U_a, V_a, W_a)$ the velocity , $P_a(t, x, y, z)$ the pressure , $\rho_a(t, x, y, z)$ the density.

Equation of state: isentropic and isothermal

$$P_a(\rho) = k\, \rho^\gamma \text{ with } k = \frac{p_a}{\rho_a^\gamma}$$

γ is set to $7/5$

- $\mathcal{A}(t,x) = \displaystyle\int_{\Omega_a} dydz$: the pseudo wetted area,
- v the mean value of the speed
  $$v(t,x) = \frac{1}{\mathcal{A}(t,x)} \int_{\Omega_a} U_a(t,x,y,z)\, dydz,$$
- $M = \overline{\rho}/\rho_0 \mathcal{A}$, $D = Mv$ rescaled air mass and air discharge.
- $c_a^2 = \dfrac{\partial p}{\partial \rho} = k\gamma \left(\dfrac{\rho_0 M}{\mathcal{A}}\right)^{\gamma-1}$, the air sound speed.

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Air layer model

$$\left\{ \begin{array}{rcl} \partial_t M + \partial_x D & = & 0 \\ \partial_t D + \partial_x \left(\dfrac{D^2}{M} + \dfrac{M}{\gamma} c_a^2 \right) & = & \dfrac{M}{\gamma} c_a^2\, \partial_x(\mathcal{A}) \end{array} \right. .$$

$\mathcal{A} + A = S$ where $S = S(x)$ denotes the pipe section

Two-layer model

$$\left\{ \begin{array}{rcl} \partial_t M + \partial_x D & = & 0 \\ \partial_t D + \partial_x \left(\dfrac{D^2}{M} + \dfrac{M}{\gamma} c_a^2 \right) & = & \dfrac{M}{\gamma} c_a^2 \, \partial_x (S - A) \\ \\ \partial_t A + \partial_x Q & = & 0 \\ \partial_t Q + \partial_x \left(\dfrac{Q^2}{A} + g I_1(x, A) \cos\theta + A \dfrac{c_a^2 \, M}{\gamma \, (S - A)} \right) & = & -gA\partial_x Z \\ & & +g I_2(x, A) \cos\theta \\ & & + \dfrac{c_a^2 \, M}{\gamma \, (S - A)} \, \partial_x A \end{array} \right.$$

$$\chi(\omega) = \chi(-\omega) \geq 0 \,, \ \int_{\mathbb{R}} \chi(\omega) d\omega = 1, \int_{\mathbb{R}} \omega^2 \chi(\omega) d\omega = 1 \quad .$$

The Gibbs equilibrium is defined by:

$$\mathcal{M}_\alpha(t, x, \xi) = \frac{A_\alpha}{b_\alpha} \chi \left(\frac{\xi - u_\alpha}{b_\alpha} \right)$$

$$b_\alpha^2 = \begin{cases} \dfrac{c_a^2}{M} & \text{if} \quad \alpha = a \\ g \dfrac{I_1(x, A)}{A} \cos\theta + \dfrac{c_a^2 \, M}{\gamma \, (S - A)} & \text{if} \quad \alpha = w \end{cases} .$$

Macroscopic unknowns:

$$(A_\alpha, Q_\alpha, u_\alpha) = \begin{cases} (A, Q, u) & \text{if} \quad \alpha = w \\ (M, D, v) & \text{if} \quad \alpha = a \end{cases}$$

- $A_\alpha = \displaystyle\int_{\mathbb{R}} \mathcal{M}_\alpha(t, x, \xi)\, d\xi$
- $Q_\alpha = \displaystyle\int_{\mathbb{R}} \xi \mathcal{M}_\alpha(t, x, \xi)\, d\xi$
- $\dfrac{Q_\alpha^2}{A_\alpha} + b_\alpha^2 A_\alpha = \displaystyle\int_{\mathbb{R}} \xi^2 \mathcal{M}_\alpha(t, x, \xi)\, d\xi$

The kinetic formulation

The couple of functions $(A_\alpha, Q_\alpha\ )$is a strong solution of the two-layer system if and only if $\mathcal{M}_\alpha$ satisfies the kinetic transport equations:

$$\partial_t \mathcal{M}_\alpha + \xi \partial_X \mathcal{M}_\alpha + \phi_\alpha \partial_\xi \mathcal{M}_\alpha = K_\alpha(t, x, \xi) \qquad \text{for } \alpha = a \text{ and } \alpha = w$$

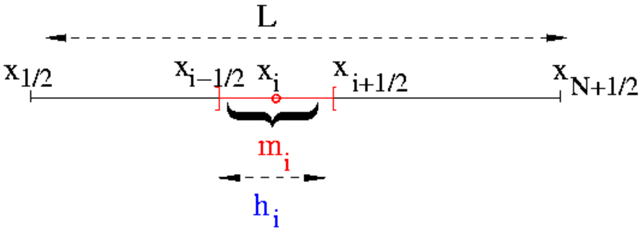
for some collision term $K_\alpha(t, x, \xi)$ which satisfies for a.e. (t, x)

$$\int_{\mathbb{R}} K_\alpha \, d\xi = 0 \, , \; \int_{\mathbb{R}} \xi \, K_\alpha d \, \xi = 0 \quad .$$

The function ϕ_α is defined by:

$$\phi_\alpha = \begin{cases} -\dfrac{c_a^2}{S-A} \partial_x (S - A) & \text{if} \quad \alpha = A \\ g \partial_x Z - g \dfrac{I_2(x, A)}{A} \cos\theta - \dfrac{M}{S-A} c_a^2 \, \partial_x \ln(A) & \text{if} \quad \alpha = W \end{cases}$$

The mesh



$W^n_{\alpha,i} = (A^n_{\alpha,i}, Q^n_{\alpha,i})$, $u^n_{\alpha,i} = \dfrac{Q^n_{\alpha,i}}{A^n_{\alpha,i}}$: approximation of the mean value of (A_α, Q_α) and the velocity u_α on m_i at time t_n.

$\phi^n_{\alpha,i}\mathbb{1}_{m_i}(X)$ approximation of ϕ_α on m_i at time t_n

Kinetic level

On $[t_n, t_{n+1}[$ the transport equation becomes:

$$\frac{\partial}{\partial t} f(t,x,\xi) + \xi \cdot \frac{\partial}{\partial x} f(t,x,\xi) = 0$$

with $f(t_n, x, \xi) = \mathcal{M}_i^n(\xi) = \mathcal{M}(A_i^n, Q_i^n, \xi)$for $x \in m_i$

Finite Volume Scheme

$$f_i^{n+1}(\xi) = \mathcal{M}_i^n(\xi) + \frac{\Delta t}{\Delta x} \xi \left(\mathcal{M}_{i+\frac{1}{2}}^-(\xi) - \mathcal{M}_{i-\frac{1}{2}}^+(\xi)\right)$$

Discontinuity of the flux at $x_{i+1/2}$ due to the jump of ϕ_α

Macroscopic level: integrate on ξ

$$\int_{\mathbb{R}} \begin{pmatrix} 1 \\ \xi \end{pmatrix} \left[f_i^{n+1}(\xi) = \mathcal{M}_i^n(\xi) + \frac{\Delta t}{\Delta x} \xi \left(\mathcal{M}^-_{i+\frac{1}{2}}(\xi) - \mathcal{M}^+_{i-\frac{1}{2}}(\xi) \right) \right] d\xi$$

$f_i^{n+1}(\xi)$ is not a Gibbs Equilibrium

$$U_i^{n+1} = \begin{pmatrix} A_i^{n+1} \\ Q_i^{n+1} \end{pmatrix} \stackrel{def}{=} \int_{\mathbb{R}} \begin{pmatrix} 1 \\ \xi \end{pmatrix} f_i^{n+1}(\xi)\, d\xi$$

$\longrightarrow \mathcal{M}_i^{n+1}$ defined without using the collision kernel

The kinetic scheme

$$U_i^{n+1} = U_i^n + \frac{\Delta t}{\Delta x}\left(F^-_{i+\frac{1}{2}} - F^+_{i-\frac{1}{2}}\right)$$

with:

$$F^\pm_{i+\frac{1}{2}} = \int_{\mathbb{R}} \xi \begin{pmatrix} 1 \\ \xi \end{pmatrix} \mathcal{M}^\pm_{i+\frac{1}{2}}(\xi)\, d\xi$$

What about the kinetic fluxes $\mathcal{M}^\pm_{i+\frac{1}{2}}(\xi)$?

The kinetic scheme

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What about the kinetic fluxes $\mathcal{M}^{\pm}_{i+\frac{1}{2}}(\xi)$?

Generalised characteristics method in the plane (x, ξ) on the equation:

$$\partial_t \mathcal{M}_\alpha + \xi \partial_X \mathcal{M}_\alpha + \phi_\alpha \partial_\xi \mathcal{M}_\alpha = K_\alpha(t, x, \xi) \quad \text{for } \alpha = a \text{ and } \alpha = w$$

At the interface $x_{i+1/2}$.

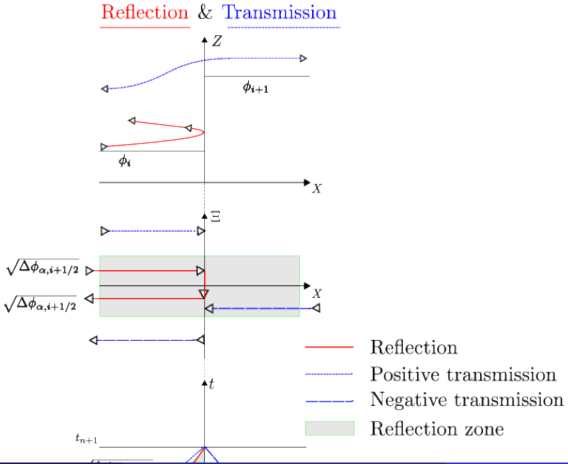
$$\Delta\phi_{\alpha,i+1/2} = \begin{cases} -\widetilde{(c_a^2)} \ln\left(\dfrac{(S - A_{i+1})}{(S - A_i)}\right) & \text{if} \quad \alpha = a \\[2em] g\,(Z_{i+1} - Z_i) - \left(\widetilde{\dfrac{c_a^2 M}{S - A}}\right)\dfrac{1}{\gamma}\ln(A_{i+1}/A_i) & \text{if} \quad \alpha = w \end{cases}.$$

Nonconservative product at the interface : $f(x, W)\partial_x W \simeq \widetilde{f}(W_{i+1} - W_i)$

$$\tilde{f} = \int_0^1 f(x_{i+1/2}, \Phi(s, W_i, W_{i+1}))\,ds \quad \Phi \text{ the characteristics line}$$

$\Delta\phi_{\alpha,i\pm1/2}$: jump condition for a particle with the kinetic speed ξ which is necessary to:

- be reflected: the particle has not enough kinetic energy $\xi^2/2$ to overpass the potential barrer with potential energy $\Delta\phi_\alpha$,
- overpass the potential barer with a positive speed,
- overpass the potential barer with a negative speed,



Expression of $M^-_{i+\frac{1}{2}}$

$$
\begin{aligned}
\mathcal{M}^-_{\alpha,i+1/2}(\xi) \quad = \quad & \overbrace{\mathbb{1}_{\{\xi>0\}}\mathcal{M}^n_{\alpha,i}(\xi)}^{\text{positive transmission}} + \overbrace{\mathbb{1}_{\left\{\xi<0,\xi^2-2g\Delta\phi_{\alpha,i+1/2}<0\right\}}\mathcal{M}^n_{\alpha,i}(-\xi)}^{\text{reflection}} \\
+ \quad & \underbrace{\mathbb{1}_{\left\{\xi<0,\xi^2-2g\Delta\phi_{\alpha,i+1/2}>0\right\}}\mathcal{M}^n_{\alpha,i+1}\left(-\sqrt{\xi^2 - 2g\Delta\phi_{\alpha,i+1/2}}\right)}_{\text{negative transmission}}
\end{aligned}
$$

Expression of $M^+_{i+\frac{1}{2}}$

$$
\begin{aligned}
\mathcal{M}^+_{\alpha,i+1/2}(\xi) &= \overbrace{\mathbb{1}_{\{\xi<0\}}\mathcal{M}^n_{\alpha,i+1}(\xi)}^{\text{negative transmission}} + \overbrace{\mathbb{1}_{\left\{\xi>0,\xi^2+2g\Delta\phi_{\alpha,i+1/2}<0\right\}}\mathcal{M}^n_{\alpha,i+1}(-\xi)}^{\text{reflection}} \\
&+ \underbrace{\mathbb{1}_{\left\{\xi>0,\xi^2+2g\Delta\phi_{\alpha,i+1/2}>0\right\}}\mathcal{M}^n_{\alpha,i}\left(\sqrt{\xi^2+2g\Delta\phi_{\alpha,i+1/2}}\right)}_{\text{positive transmission}}
\end{aligned}
$$

We choose:

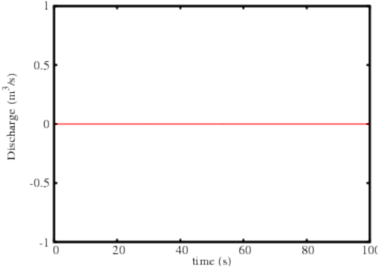
$$\chi(\omega) = \frac{1}{2\sqrt{3}} \mathbb{1}_{[-\sqrt{3},\sqrt{3}]}(\omega)$$

Properties of the kinetic scheme

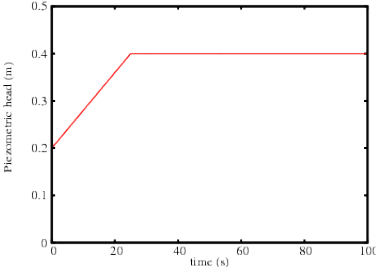
We assume a CFL condition. Then

- the kinetic scheme keeps the wetted area $A^n_{\alpha,i}$ positive,
- the kinetic scheme preserves the still water/air steady state,
- Drying and flooding are treated.

Horizontal circular pipe : $L = 100\ m\ D = 2\ m$.
Inital steady state: $Q_w = 0\ m^3/s$ and $y_w = 0.2\ m$.
Upstream water height is increasing in 25 s at $y_w = 0.4\ m$
At downstream and upstream :
$\rho_a = 1.29349\ kg/m^3$ or $1.29349\ 10^{-2}\ kg/m^3$ and $Q_a = 0\ m^3/s$

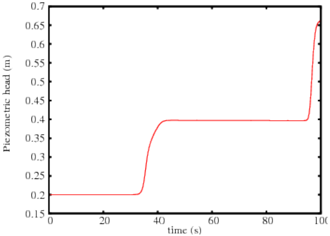


(a) Downstream Air and Water discharge

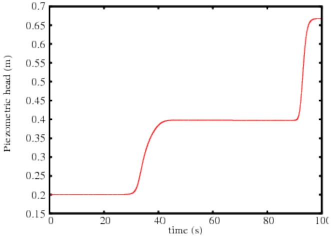


(b) Upstream Water piezomeric head

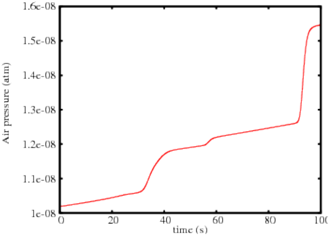
Piezomeric head for $\rho_a = 1.29349\ 10^{-2}\ kg/m^3$ at the middle of the pipe



(c) Single fluid

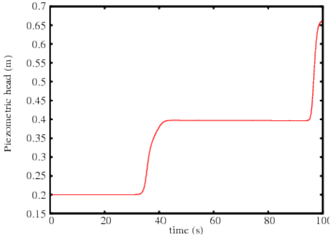


(d) Piezometric head of water

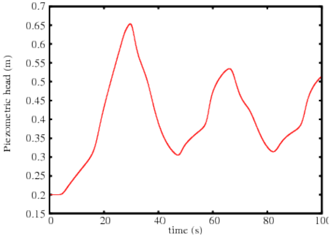


(e) Air pressure

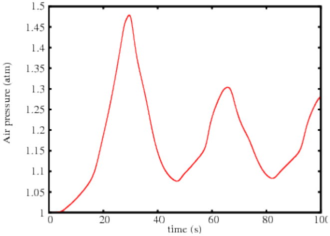
Piezomeric head for $\rho_a = 1.29349\ kg/m^3$ at the middle of the pipe



(f) Single fluid



(g) Piezometric head of water



(h) Air pressure

- **Single fluid**

- $\boldsymbol{\rho_a = 1.29349\ 10^{-2}\ \mathrm{kg/m^3}}$

- $\boldsymbol{\rho_a = 1.29349\ \mathrm{kg/m^3}}$

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Conclusion

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- Good properties : total entropy, well-balanced.

To do

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- Condensation/Evaporation

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Thank you for your attention