Mixed flows in closed water pipes

A kinetic approach

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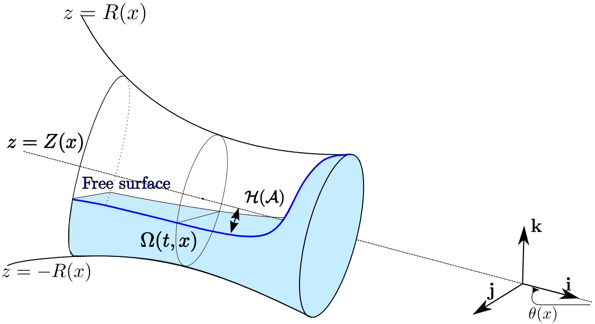
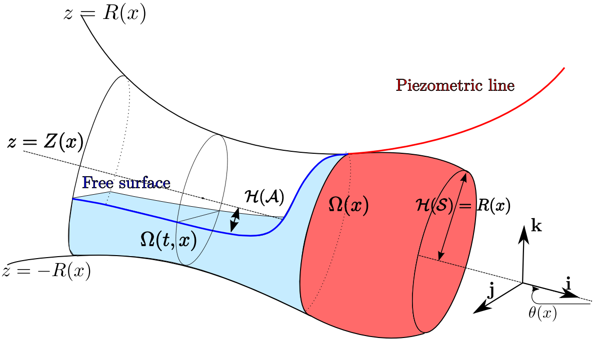
- Free surface (FS) area : only a part of the section is filled.

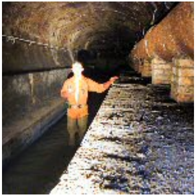


- Free surface (FS) area : only a part of the section is filled.
- Pressurized (P) area : the section is completely filled.

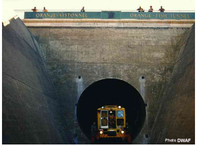


Some closed pipes

a forced pipe

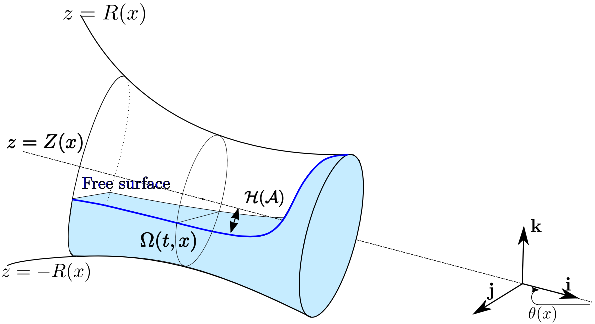


a sewer in Paris



The Orange-Fish Tunnel (in Canada)

$$\begin{aligned} \operatorname{div}(\rho_0\, U) &= 0 \\ \partial_t(\rho_0\, U) + \rho_0\, U \cdot \nabla(\rho_0\, U) + \nabla P &= \rho_0 F \end{aligned}$$



The domain $\Omega_F(t)$ of the flow at time t : the union of sections $\Omega(t,x)$ orthogonal to some plane curve $\mathcal{C}$ lying in $(O,\mathbf{i},\mathbf{k})$ following main flow axis. $\omega = (x, 0, b(x))$ in the cartesian reference frame $(O,\mathbf{i},\mathbf{j},\mathbf{k})$ where $\mathbf{k}$ follows the vertical direction; $b(x)$ is then the elevation of the point $\omega(x, 0, b(x))$ over the plane $(O,\mathbf{i},\mathbf{j})$

Curvilinear variable defined by:

$$X = \int_{x_0}^{x} \sqrt{1 + (b'(\xi))^2} d\xi$$

where x_0 is an arbitrary abscissa. $Y = y$ and we denote by Z the $\mathbf{B}$-coordinate of any fluid particle M in the Serret-Frenet reference frame $(\mathbf{T},\mathbf{N},\mathbf{B})$ at point $\omega(x, 0, b(x))$.

1. write the Euler equations in a curvilinear reference frame,
2. $\epsilon = H/L$ with H (the height) and L (the length) and take $\epsilon = 0$ in the Euler curvilinear equations,
3. the conservative variables $A(t, X)$: the wet area, $Q(t, X)$ the discharge defined by

$$A(t,X) = \int_{\Omega(t,X)} dYdZ, \quad Q(t,X) = A(t,X)\overline{U}$$

$$\overline{U}(t,X) = \frac{1}{A(t,X)} \int_{\Omega(t,X)} U(t,X)\, dYdZ.$$

4. approximation :$\overline{U^2} \approx \overline{U}\,\overline{U}$ and $\overline{U\,V} \approx \overline{U}\,\overline{V}$.

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$$\left\{\begin{array}{lcl} \partial_t A + \partial_X Q & = & 0 \\ \partial_t Q + \partial_X \left(\dfrac{Q^2}{A} + g I_1(X,A)\cos\theta\right) & = & g I_2(X,A)\cos\theta - gA\sin\theta \\ & & -gA\overline{Z}(X,A)(\cos\theta)' \end{array}\right. \qquad (1)$$

$I_1(X,A) = \displaystyle\int_{-R}^{h} (h-Z)\sigma\, dZ$: the hydrostatic pressure term

$I_2(X,A) = \displaystyle\int_{-R}^{h} (h-Z)\partial_X\sigma\, dZ$: the pressure source term

$\widetilde{p} = \rho_0(h(t,X) - Z)\cos\theta$: the hydrostatic pressure.

$\overline{Z} = \displaystyle\int_{\Omega(t,X)} Z\, dY\, dZ$: the center of mass

We add the Manning-Strickler friction term of the form

$$S_f(A,U) = K(A)U|U|\,.$$

$$\partial_t \rho + \mathrm{div}(\rho \mathbf{U}) = 0, \tag{2}$$

$$\partial_t(\rho \mathbf{U}) + \mathrm{div}(\rho \mathbf{U} \otimes \mathbf{U}) + \nabla p = \mathbf{F}, \tag{3}$$

Linearized pressure law:

$$p = p_a + \frac{\rho - \rho_0}{\beta \rho_0}$$

$$c = \frac{1}{\sqrt{\beta \rho_0}}$$

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3. the conservative variables $A(t, X)$: the wet equivalent area, $Q(t, X)$ the equivalent discharge defined by

$$A = \frac{\rho}{\rho_0} S \,,\; Q = A\overline{U}$$

$$\overline{U}(t, X) = \frac{1}{S(,X)} \int_{S(X)} U(t, X)\; dYdZ.$$

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$$
\left\{
\begin{array}{rcl}
\partial_t(A) + \partial_X(Q) & = & 0 \\
\partial_t(Q) + \partial_X\left(\dfrac{Q^2}{A} + c^2 A\right) & = & -gA\sin\theta - gA\overline{Z}(X,S)(\cos\theta)' \\
 & & +c^2 A \dfrac{S'}{S}
\end{array}
\right. \qquad (4)
$$

$c^2 A$: the pressure term
$c^2 A \frac{S'}{S}$: the pressure source term due to geometry changes
$gA\overline{Z}(X,S)(\cos\theta)'$: the pressure source term due to the curvature
$\overline{Z}$: the center of mass.
We add the Manning-Strickler friction term of the form

$$S_f(A,U) = K(A)U|U|\,.$$

$$\begin{cases} \partial_t(A) + \partial_x(Q) & = 0 \\ \partial_t(Q) + \partial_x\left(\dfrac{Q^2}{A} + p(x,A,S)\right) & = -g\,A\,\dfrac{d}{dx}Z(x) \\ & \quad + Pr(x,A,S) \\ & \quad - G(x,A,S) \\ & \quad - g\,A\,K(x,S)\,u\,|u| \end{cases} .$$

- $A = \dfrac{\rho}{\rho_0} S$: wet equivalent area,
- $Q = A\,u$: discharge,
- S the physical wet area.

The pressure is $p(x,A,S) = c^2\,(A - S) + g\,I_1(x,S)\cos\theta$.

- The pressure source term:

$$Pr(x, A, S) = \left(c^2 \left(A/S - 1\right)\right) \frac{d}{dx} S + g\, I_2(x, S) \cos\theta,$$

- the $z-$coordinate of the center of mass term:

$$G(x, A, S) = g\, A\, \overline{Z}(x, S) \frac{d}{dx} \cos\theta,$$

- the friction term:

$$K(x, S) = \frac{1}{K_s^2\, R_h(S)^{4/3}}.$$

  - $K_s > 0$ is the Strickler coefficient,
  - $R_h(S)$ is the hydraulic radius.

[BEG09] *C. Bourdarias, M. Ersoy and S. Gerbi.* A model for unsteady mixed flows in non uniform closed water pipes and a well-balanced finite volume scheme. *IJFV , 2009.*

Mathematical properties

- The PFS system is strictly hyperbolic for $A(t,x) > 0$.
- For smooth solutions, the mean velocity $u = Q/A$ satisfies

$$\partial_t u + \partial_x \left(\frac{u^2}{2} + c^2 \ln(A/S) + g\,\mathcal{H}(S)\cos\theta + g\,Z \right) = -g\,K(x,S)\,u\,|u| \ .$$

  and $u = 0$ reads: $c^2 \ln(A/S) + g\,\mathcal{H}(S)\cos\theta + g\,Z = 0$.
- It admits a mathematical entropy

$$E(A,Q,S) = \frac{Q^2}{2A} + c^2 A \ln(A/S) + c^2 S + g\overline{Z}(x,S)\cos\theta + gAZ$$

  which satisfies the entropy inequality

$$\partial_t E + \partial_x \left(E\,u + p(x,A,S)\,u \right) = -g\,A\,K(x,S)\,u^2\,|u| \leqslant 0$$

With

$$\chi(\omega) = \chi(-\omega) \geq 0 \ , \ \int_{\mathbb{R}} \chi(\omega) d\omega = 1, \int_{\mathbb{R}} \omega^2 \chi(\omega) d\omega = 1 \ ,$$

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we define the Gibbs equilibrium

$$\mathcal{M}(t,x,\xi) = \frac{A}{c(A)} \chi\left(\frac{\xi - u(t,x)}{c(A)}\right)$$

with

$$c(A) = \sqrt{g \frac{I_1(x,A)}{A} \cos\theta} \text{ in the FS zones and,}$$

$$c(S) = \sqrt{g \frac{I_1(x,S)}{S} \cos\theta + c^2} \text{ in the P zones.}$$

We have the macroscopic-microscopic relations:

$$A = \int_{\mathbb{R}} \mathcal{M}(t, x, \xi)\, d\xi$$

$$Q = \int_{\mathbb{R}} \xi \mathcal{M}(t, x, \xi)\, d\xi$$

$$\frac{Q^2}{A} + Ac(A)^2 = \int_{\mathbb{R}} \xi^2 \mathcal{M}(t, x, \xi)\, d\xi$$

The Kinetic Formulation

(A, Q) is a strong solution of PFS-System if and only if $\mathcal{M}$ satisfies the kinetic transport equation:

$$\partial_t \mathcal{M} + \xi \cdot \partial_x \mathcal{M} - g\Phi(x, A, S)\, \partial_\xi \mathcal{M} = K(t, x, \xi)$$

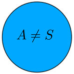
for some collision term $K(t, x, \xi)$ which satisfies for a.e. (t, x)

$$\int_{\mathbb{R}} K\, d\xi = 0\;, \;\int_{\mathbb{R}} \xi\, K d\,\xi = 0$$

Φ takes into account all the source terms.

[P02] *B. Perthame.* Kinetic formulation of conservation laws. Oxford University Press. *Oxford Lecture Series in Mathematics and its Applications, Vol 21, 2002.*

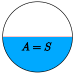
The source terms

- If 

, Φ reads:

$$\overbrace{\frac{d}{dx}Z - \frac{c^2}{g}\frac{d}{dx}\ln(S)}^{\text{Conservative}} + \overbrace{\overline{Z}(x,S)\frac{d}{dx}\cos\theta}^{\text{Non conservative product}}$$

$$+\frac{d}{dx}\int_x K(x,S)u|u|\,dx$$

- If 

, Φ reads:

$$\overbrace{\frac{d}{dx}Z}^{\text{Conservative}} + \overbrace{\frac{\gamma(x,A)\cos\theta}{A}\frac{d}{dx}\ln(A) + \overline{Z}(x,A)\frac{d}{dx}\cos\theta}^{\text{Non conservative product}}$$

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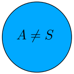
Back

- If 

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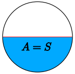
- If 

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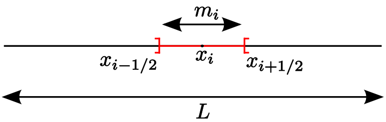
The mesh and the unknowns



Geometric terms and unknowns are piecewise constant approximations on the cell m_i at time t_n:

- Geometric terms
  - S_i, $\cos\theta_i$
- Macroscopic unknowns
  - $\mathbf{W}_i^n = (A_i^n, Q_i^n)$, $u_i^n = \dfrac{Q_i^n}{A_i^n}$
- Microscopic unknown
  - $\mathcal{M}_i^n(\xi) = \dfrac{A_i^n}{c_i^n}\chi\left(\dfrac{\xi - u_i^n}{c_i^n}\right)$

Consequently Φ_i^n is null on m_i.

Indeed, we have:

- $\dfrac{d}{dx}(\mathbb{1}_{m_i} Z) = 0,$
- $\dfrac{d}{dx}(\ln(\mathbb{1}_{m_i} S)) = 0,$
- $\dfrac{d}{dx}(\mathbb{1}_{m_i} \cos\theta) = 0,$
- $\dfrac{d}{dx}\displaystyle\int_x K(x,S)u|u|\,dx = 0$ Go

[PS01] *B. Perthame and C. Simeoni*. A kinetic scheme for the Saint-Venant system with a source term. *Calcolo, Vol 38(4) 201–231, 2001*

Neglecting the collision term, the transport equation reads on $[t_n, t_{n+1}[\times m_i$:

$$\frac{\partial}{\partial t} f + \xi \cdot \frac{\partial}{\partial x} f = 0$$

with $f(t_n, x, \xi) = \mathcal{M}_i^n(\xi)$ for $x \in m_i$ and thus it is discretised on m_i as:

$$f_i^{n+1}(\xi) = \mathcal{M}_i^n(\xi) + \frac{\Delta t^n}{\Delta x} \xi \left(\mathcal{M}_{i+\frac{1}{2}}^-(\xi) - \mathcal{M}_{i-\frac{1}{2}}^+(\xi) \right) ,$$

Although f_i^{n+1} is not a Gibbs equilibrium, we have :

$$\mathbf{W}_i^{n+1} = \begin{pmatrix} A_i^{n+1} \\ Q_i^{n+1} \end{pmatrix} \stackrel{def}{:=} \int_{\mathbb{R}} \begin{pmatrix} 1 \\ \xi \end{pmatrix} f_i^{n+1}(\xi)\, d\xi$$

$\longrightarrow \mathcal{M}_i^{n+1}$ defined without using the collision kernel : it is a way to perform all collisions at once

Finally the kinetic scheme reads:

$$\mathbf{W}_i^{n+1} = \mathbf{W}_i^n + \frac{\Delta t^n}{\Delta x}\left(F^-_{i+\frac{1}{2}} - F^+_{i-\frac{1}{2}}\right)$$

with the interface fluxes

$$F^\pm_{i+\frac{1}{2}} = \int_{\mathbb{R}} \xi \begin{pmatrix} 1 \\ \xi \end{pmatrix} \mathcal{M}^\pm_{i+\frac{1}{2}}(\xi)\, d\xi$$

where the microscopic fluxes are defined following e.g. [BEG09b, PS01]:

[BEG09b] *C. Bourdarias and M. Ersoy and S. Gerbi.* A kinetic scheme for pressurised flows in non uniform closed water pipes. *Monografias de la Real Academia de Ciencias de Zaragoza, Vol 31 1–20, 2009.*

The microscopic fluxes are given by

Expression of $\mathcal{M}^{-,n}_{i+1/2}$, $\mathcal{M}^{+,n}_{i+1/2}$

$$\begin{aligned}
\mathcal{M}^{-,n}_{i+1/2} &= \overbrace{\mathbb{1}_{\{\xi>0\}}\mathcal{M}^n_i(\xi)}^{\text{positive transmission}} + \overbrace{\mathbb{1}_{\left\{\xi<0,\xi^2-2g\Delta\phi^n_{i+1/2}<0\right\}}\mathcal{M}^n_i(-\xi)}^{\text{reflection}} \\
&+ \underbrace{\mathbb{1}_{\left\{\xi<0,\xi^2-2g\Delta\phi^n_{i+1/2}>0\right\}}\mathcal{M}^n_{i+1}\left(-\sqrt{\xi^2-2g\Delta\phi^n_{i+1/2}}\right)}_{\text{negative transmission}} \\
\mathcal{M}^{+,n}_{i+1/2} &= \overbrace{\mathbb{1}_{\{\xi<0\}}\mathcal{M}^n_{i+1}(\xi)}^{\text{negative transmission}} + \overbrace{\mathbb{1}_{\left\{\xi>0,\xi^2+2g\Delta\phi^n_{i+1/2}<0\right\}}\mathcal{M}^n_{i+1}(-\xi)}^{\text{reflection}} \\
&+ \underbrace{\mathbb{1}_{\left\{\xi>0,\xi^2+2g\Delta\phi^n_{i+1/2}>0\right\}}\mathcal{M}^n_{i}\left(\sqrt{\xi^2+2g\Delta\phi^n_{i+1/2}}\right)}_{\text{positive transmission}}
\end{aligned} \tag{5}$$

The potential barrier $\Delta\phi^n_{i\pm 1/2}$ has the following expression:

$$\Delta\phi^n_{i+1/2} = \begin{cases} \left[\left[Z + \int_x K(x,S)u|u|\,dx\right]\right]_{i+1/2} \\ -\dfrac{c^2}{g}\,[[\ln(S)]]_{i+1/2} \\ +[[\cos\theta]]_{i+1/2}\displaystyle\int_0^1 \overline{Z}(s,\psi_S(s))ds & \text{if } E_i^n = 1 \\ \\ \left[\left[Z + \int_x K(x,A)u|u|\,dx\right]\right]_{i+1/2} \\ -[[A]]_{i+1/2}\displaystyle\int_0^1 \frac{\gamma(s,\psi_A(s))}{\psi_A(s)}(\psi_{\cos\theta})ds \\ +[[\cos\theta]]_{i+1/2}\displaystyle\int_0^1 \overline{Z}(s,\psi_A(s))ds & \text{if } E_i^n = 0 \end{cases}$$

where ψ_A (resp. ψ_S) is the straight lines path connecting the left state A_i (resp. S_i) to the right one A_{i+1} (resp. S_{i+1}).

The term $\xi^2 \pm 2g\Delta\phi^n_{i+1/2}$ is the jump condition for a particle with the kinetic speed ξ which is necessary to

- be reflected: this means that the particle has not enough kinetic energy $\xi^2/2$ to overpass the potential barrier (reflection in (5)),
- overpass the potential barrier with a positive speed (positive transmission in (5)),
- overpass the potential barrier with a negative speed (negative transmission in (5)).

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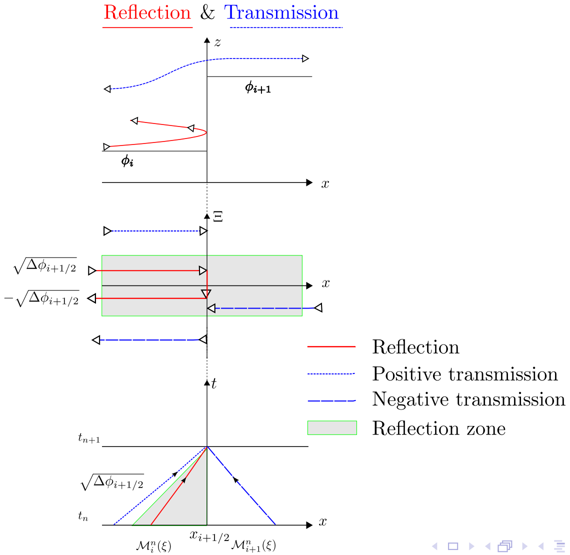
Reflection & Transmission
z
ϕ_{i+1}
ϕ_i
x
Ξ
$\sqrt{\Delta\phi_{i+1/2}}$
$-\sqrt{\Delta\phi_{i+1/2}}$
x
t
t_{n+1}
$\sqrt{\Delta\phi_{i+1/2}}$
t_n
x
$\mathcal{M}_i^n(\xi)$
$x_{i+1/2}$
$\mathcal{M}_{i+1}^n(\xi)$
Reflection
Positive transmission
Negative transmission
Reflection zone

Outline

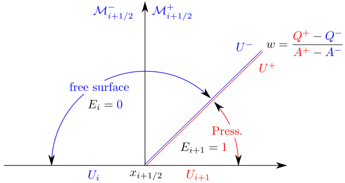

Figure: Free Surface / Pressurised

We have 5 unknowns : U^+, U^-, w.
5 equations :

1. 2 jumps conditions
2. 2 relations to compute $\mathcal{M}^{+,n}_{i+1/2}$
3. Conservation of energy

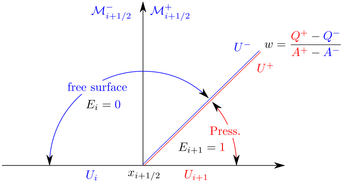

Figure: Free Surface / Pressurised

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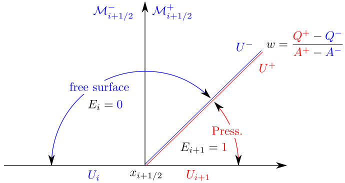

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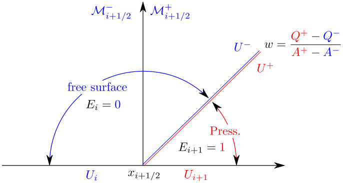

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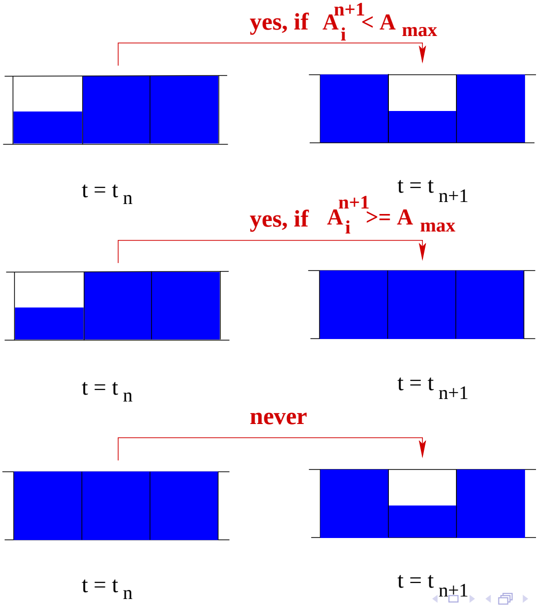
State update



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[ABP00] *E. Audusse and M-0. Bristeau and B. Perthame*. Kinetic schemes for Saint-Venant equations with source terms on unstructured grids. *INRIA Report RR3989, 2000.*

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We assume a CFL condition. Then

Properties of the numerical scheme

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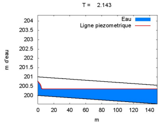
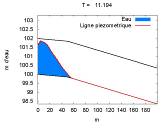
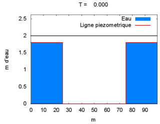
- A water-hammer test



- An injection test



- A double dam break



Conclusion

- Easy implementation of source terms
- Very good agreement for uniform case
- Drying and flooding area are computed

Perspective

- Air entrainment treated as a bilayer fluid flow (in progress).
- Diphasic approach to take into account air entrapment, evaporation/condensation and cavitation.
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Thank you for your attention