Habilitation à Diriger des Recherches

FROM HYDROSTATIC TO NON-HYDROSTATIC MODELS IN FLUID MECHANICS: MODELING, MATHEMATICAL AND NUMERICAL ANALYSIS, AND COMPUTATIONAL FLUID DYNAMICS

Mehmet Ersoy

2020, 01 December, La Garde, France

Outline of the talk

- Fluids are everywhere ! ! !
  - Atmosphere/land : weather, rain, storms, flooding, water ressources, *etc.*

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  - Atmosphere/land
  - Underground : sandy beaches, underground networks, sewers, rivers, phreatic (groundwater), erosion, sedimentation *etc.*

- Fluids are everywhere ! ! !
  - Atmosphere/land
  - Underground
  - Sea/ocean/Channel : maritime, navigation, erosion, sedimentation, tsunamis, breaking waves and even sounds like health *etc.*

Nazare

- Fluids are everywhere ! ! !
  - Atmosphere/land
  - Underground
  - Sea/ocean/Channel
- Multiple scales, non trivial interactions/coupling yielding to hydrostatic to non hydrostatic phenomenon

- Introducing characteristic scales :
  - length L
  - width l
  - height H

- Introducing characteristic scales : L, l and H
- Introducing aspect ratio numbers :
  - $\varepsilon_z = \dfrac{H}{L}$ following the depth
  - $\varepsilon_y = \dfrac{l}{L}$ following the width

- Introducing characteristic scales : L, l and H
- Introducing aspect ratio numbers : $\varepsilon_z = \dfrac{H}{L}$ and $\varepsilon_y = \dfrac{l}{L}$
- One can reduce the initial model (Navier-Stokes or Euler equations)
  - 3D-2D depth averaged model reduction if

$$\varepsilon_z \ll 1 \text{ and } \varepsilon_y \approx 1$$

  - 3D-1D section averaged model reduction if

$$\varepsilon_z \approx \varepsilon_y \ll 1$$

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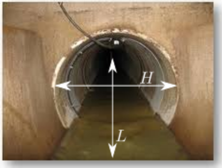
- Opposite to DNS, model reduction $\to$ to decrease the computational cost

- Introducing characteristic scales : L, l and H
- Introducing aspect ratio numbers :
- One can reduce the initial model (Navier-Stokes or Euler equations)
- Opposite to DNS, model reduction → to decrease the computational cost
- Some applications :

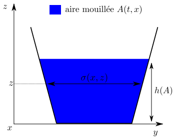


Outline

1. Hydrostatic models, applications and limits
   - Hydrostatic models
   - Application to tsunamis propagation
2. Non-hydrostatic models and applications
   - Historical background and motivations
   - Toward the first dispersive section-averaged model
3. Concluding remarks and perspectives

SV equations

- for closed water pipes/channels/rivers



$$
\begin{cases}
\partial_t A + \partial_x Q = 0, \\
\partial_t Q + \partial_x \left(\dfrac{Q^2}{A} + g I_1(x,A) \right) = g I_2(x,A)
\end{cases}
$$

with

$A(t,x)$, $Q(t,x)$, g, $h = \eta - d$	:	wet area, discharge, gravity
$I_1(x,A) = \int_d^\eta \sigma(x,z)(\eta - z)dz$	:	hydrostatic pressure
$I_2(x,A) = \int_d^\eta \dfrac{\partial}{\partial x}\sigma(x,z)(\eta - z)dz$	:	hydrostatic pressure source

C. Bourdarias, **M. Ersoy**, S. Gerbi.
A kinetic scheme for pressurized flows in non uniform pipes.
Monografias de la Real Academia de Ciencias, 2009.

C. Bourdarias, **M. Ersoy**, S. Gerbi.
A model for unsteady mixed flows in non uniform closed water pipes and a well-balanced finite volume scheme.
International Journal on Finite Volumes, 2009.

C. Bourdarias, **M. Ersoy**, S. Gerbi.
A kinetic scheme for transient mixed flows in non uniform closed pipes : a global manner to upwind all the source terms.
Journal of Scientific Computing, 2011.

C. Bourdarias, **M. Ersoy**, S. Gerbi.
Unsteady mixed flows in non uniform closed water pipes : a Full Kinetic Appraoch.
Numerische Mathematik, 2014.

SV equations

- for closed water pipes/channels/rivers

$$\left\{\begin{array}{l}\partial_t A + \partial_x Q = 0,\\ \partial_t Q + \partial_x \left(\dfrac{Q^2}{A} + gI_1(x,A)\right) = gI_2(x,A) - gAK(x,Q/A)\end{array}\right.$$

with $K(x,u) = \dfrac{K_0(u)}{A} \displaystyle\int_{\Gamma_b(x,t)} ds$ where

- $K_0(u) = C_l + C_t\,|u|$
- $A/\displaystyle\int_{\Gamma_b}(x,t)ds$ is the so-called hydraulic radius

- for closed water pipes/channels/rivers including friction

M. Ersoy.
Dimension reduction for incompressible pipe and open channel flow including friction.
Applications of Mathematics, 2015.

M. Ersoy.
Dimension reduction for compressible pipe flows including friction.
Asymptotic Analysis, 2016.

SV equations

- for closed water pipes/channels/rivers
- for closed water pipes/channels/rivers including friction
- for urban/overland flows including precipitation and recharge

$$\begin{cases} \partial_t h + \partial_x q = S := R - I, \\ \partial_t q + \partial_x \left(\dfrac{q^2}{A} + g\dfrac{h^2}{2} \right) = -gh\partial_x Z + S\dfrac{q}{h} - \left(k_+(R) + k_-(I) + k_0\left(\dfrac{q}{h}\right)\right)\dfrac{q}{h} \end{cases}$$

with $h(t,x)$, $q(t,x)$: water height, discharge
$k_\pm$: friction generated from precipitation and infiltration

where I can be driven by the solution of the Richards' equation.

M. Ersoy, O. Lakkis, P. Townsend.
A Saint-Venant shallow water model for overland flows with precipitation and recharge.
Mathematical and Computational Applications, Natural Sciences, 2020.

J.-B. Clément, **M. Ersoy**, F. Golay, and D. Sous.
Discontinuous galerkin method for steady-state richards equation.
Topical Problems of Fluid Mechanics, 2019

J.-B. Clément, **M. Ersoy**, F. Golay, and D. Sous.
Adaptive discontinuous galerkin method for richards equation.
Topical Problems of Fluid Mechanics, 2020

J.-B. Clément, **M. Ersoy**, F. Golay, and D. Sous.
An adaptive strategy for discontinuous Galerkin simulations of Richards' equation.
Preprint, 2020

J.-B. Clément, D. Sous, F. Golay, and **M. Ersoy**.
Wave-driven Ground- water Flows in Sandy Beaches : A Richards Equation-based Model.
Journal of Coastal Research, 2020

SV equations

- for closed water pipes/channels/rivers
- for closed water pipes/channels/rivers including friction
- for urban/overland flows including precipitation and recharge
- for tsunamis propagation

$$\left\{\begin{array}{l} \partial_t h + \mathsf{div}(h\overline{u}) = 0, \\ \partial_t (h\overline{u}) + \mathsf{div}\left(h\overline{u}\otimes\overline{u} + g\dfrac{h^2}{2}I\right) = -gh\nabla Z, \end{array}\right.$$

with $\overline{u}(t,x)\in\mathbb{R}^2$: depth averaged velocity

K. Pons, **M. Ersoy**.
Adaptive mesh refinement method. Part 1 : Automatic thresholding based on a distribution function.
SEMA SIMAI Springer Series, Partial Differential Equations : Ambitious Mathematics for Real-Life Applications, D. Donatelli and C. Simeoni Editors, 2020

K. Pons, **M. Ersoy** , F. Golay and R. Marcer.
Adaptive mesh refinement method. Part 2 : Application to tsunamis propagation.
SEMA SIMAI Springer Series, Partial Differential Equations : Ambitious Mathematics for Real-Life Applications, D. Donatelli and C. Simeoni Editors, 2020

Outline

- Tsunamis are water waves that start in the deep ocean : H is huge

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- But, the wavelength λ of the tsunami is huge as well (200 km)

  - Change λ in L in the derivation $\rightarrow$ shallow water models
  - Dynamics of tsunamis are "essentially" governed by the shallow water equations.
  - Consequence phase speed of propagation $v_\phi \approx \sqrt{gH}$ (H ocean depth), either $v_\phi \approx 600$ km/h for $H = 3$km.
  - Thus, λ in L in the derivation $\rightarrow$ shallow water models : justify the use of Saint-Venant equations for some tsunamis.

- Tsunamis are water waves that start in the deep ocean : H is huge
- But, the wavelength λ of the tsunami is huge as well (200 km) $\rightarrow$ shallow water models
- Large scale numerical simulation $\rightarrow$ Adaptive strategy : principle.
  - To cluster more grid points in the regions with large solution variations, singularities or oscillations.
  - To get "Optimal mesh" : a mesh on which some physical or computational quantities (gradient, error, *etc.*) are approximately the same on each element (equi-distribution strategy)

- Tsunamis are water waves that start in the deep ocean : H is huge
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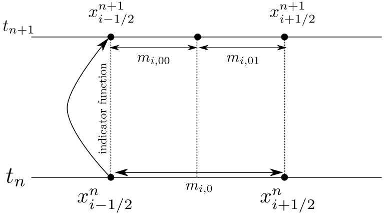
- Large scale numerical simulation $\rightarrow$ Adaptive strategy : methods.
  - h-method (Adaptive Mesh Refinement method) involves automatic refinement or coarsening of the spatial mesh based on a posteriori error estimates, error indicators or heuristic indicators.



h-method

- Tsunamis are water waves that start in the deep ocean : H is huge
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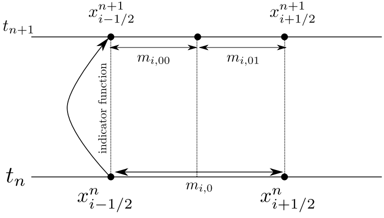
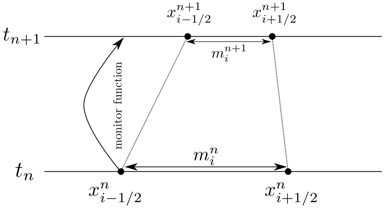
- Large scale numerical simulation $\rightarrow$ Adaptive strategy : methods.
  - h-method (Adaptive Mesh Refinement method) involves automatic refinement or coarsening of the spatial mesh based on a posteriori error estimates, error indicators or heuristic indicators.
  - r-method (Moving Mesh Method) relocates grid points in a mesh having a fixed number of nodes.



h-method



r-method

We focus on general non linear hyperbolic conservation laws

$$\begin{cases} \dfrac{\partial \boldsymbol{w}}{\partial t} + \dfrac{\partial \boldsymbol{f}(\boldsymbol{w})}{\partial x} = 0,\ (x,t) \in \mathbb{R} \times \mathbb{R}^+ \\ \boldsymbol{w}(x,0) = \boldsymbol{w}_0(x),\ x \in \mathbb{R} \end{cases}$$

$\boldsymbol{w} \in \mathbb{R}^d$	:	vector state,
$\boldsymbol{f}$	:	flux governing the physical description of the flow.

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Weak solutions satisfy

$$S = \frac{\partial s(\boldsymbol{w})}{\partial t} + \frac{\partial \psi(\boldsymbol{w})}{\partial x} \left\{ \begin{array}{lll} = & 0 & \text{for smooth solution} \\ = & 0 & \text{across rarefaction} \\ < & 0 & \text{across shock} \end{array} \right.$$

where (s, ψ) stands for a convex entropy-entropy flux pair

Entropy inequality $\simeq$ "smoothness indicator"

M. Ersoy, F. Golay, L. Yushchenko.
Adaptive multi scale scheme based on numerical density of entropy production for conservation laws
Central European Journal of Mathematics, Springer, 2013

L. Yushchenko, F. Golay, **M. Ersoy**.
Entropy production and mesh refinement – Application to wave breaking.
Mechanics & Industry, EDP Sciences, 2015

F. Golay, **M. Ersoy**, L. Yushchenko, D. Sous.
Block-based adaptive mesh refinement scheme using numerical density of entropy production for three-dimensional two-fluid flows.
International Journal of Computational Fluid Dynamics, 2015.

T. Altazin, **M. Ersoy**, F. Golay, D. Sous, L. Yushchenko.
Numerical investigation of BB-AMR scheme using entropy production as refinement criterion.
International Journal of Computational Fluid Dynamics, 2016.

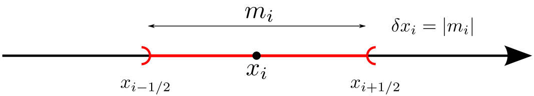

Figure – a cell m_i

Finite volume approximation :

$$\boldsymbol{w}_i^{n+1} = \boldsymbol{w}_i^n - \frac{\delta t_n}{\delta x_i}\left(\boldsymbol{F}_{i+1/2}^n - \boldsymbol{F}_{i-1/2}^n\right)$$

with

$$\boldsymbol{w}_i^n \simeq \frac{1}{\delta x_i}\int_{m_i} \boldsymbol{w}\left(x, t_n\right)\, dx \text{ and } \boldsymbol{F}_{i+1/2}^n \approx \frac{1}{\delta t}\int_{m_i} \boldsymbol{f}(t, w(x_{i+1/2}, t))\, dx$$

The numerical density of entropy production :

$$S_i^n = \frac{s_i^{n+1} - s_i^n}{\delta t_n} + \frac{\psi_{i+1/2}^n - \psi_{i-1/2}^n}{\delta x_i} \lessapprox 0$$

Assume that $\boldsymbol{w}_i^n$ is given for all i and $S := |S|$ is a given mesh refinement criterion. Then,

- Compute $S_{i_b}^n$
- Compare to $S_m = \dfrac{1}{|\Omega|} \sum_{i_b} S_{i_b}^n$

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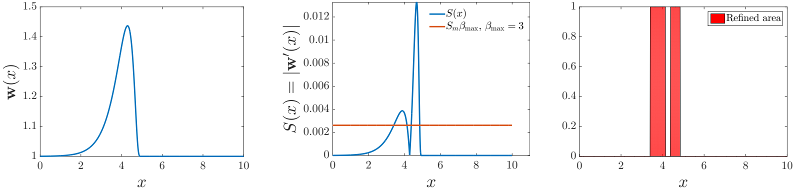
    - if $S_{i_b}^n > \alpha_{\max} = S_m \beta_{\max}$, the cell is refined and split

where $0 < \beta_{\max} \leq 1$ is user calibrated mesh refinement threshold.



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  - if $S_{i_b}^n > \alpha_{\max} = S_m \beta_{\max}$, the cell is refined and split
  - if $S_{i_{b0}}^n < \alpha_{\min} = S_m \beta_{\min}$ and $S_{i_{b1}}^n < \alpha_{\min}$, the cell is coarsened into a cell m_{i_b}

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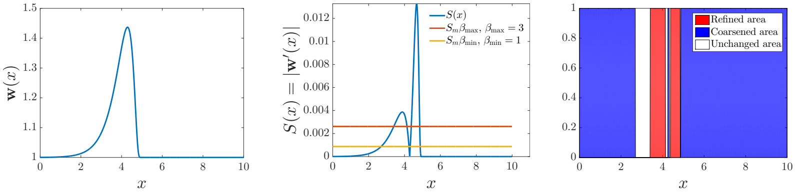

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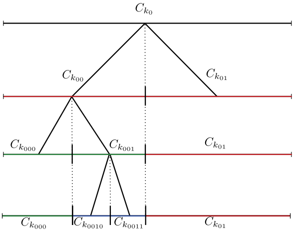
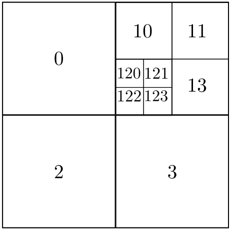
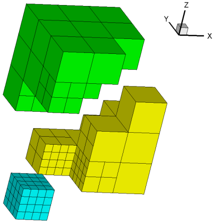


Dyadic tree



quadtree



octree

Principle of AMR methods

Assume that $\boldsymbol{w}_i^n$ is given for all i and $S := |S|$ is a given mesh refinement criterion. Then,

- Compute $S_{i_b}^n$
- Compare to $S_m = \dfrac{1}{|\Omega|}\sum_{i_b} S_{i_b}^n$
- $\beta_{\min}$ and $\beta_{\max}$ might be the critical weakness of the AMR methods, or equivalently $\alpha \mathrm{min}$ and $\alpha \mathrm{max}$

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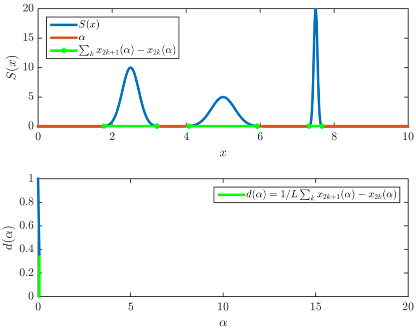
- $\beta_{\min}$ and $\beta_{\max}$ might be the critical weakness of the AMR methods, or equivalently $\alpha\min$ and $\alpha\max$

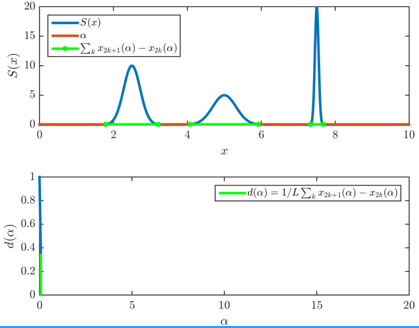
In what follows assume that $\alpha\min = \alpha\max$ for the sake of simplicity

- Assumptions and notations
  - S is smooth and has p local maxima.
  - $S(0) = S(L) = S'(0) = S'(L) = 0$
  - $0 < S_\infty = \max_{x \in (0,L)} S(x) < \infty$



- Assumptions and notations
- One can define the distribution $d := \text{meas}\{S(x) > \alpha\}$

$$\alpha \in [0, S_\infty] \mapsto d(\alpha) := \begin{cases} 1 & \text{if} \quad \alpha = 0 \, , \\ \dfrac{1}{L} \displaystyle\sum_{k=1}^{p_\alpha} x_{2k+1}(\alpha) - x_{2k}(\alpha) & \text{if} \quad 0 < \alpha < S_\infty \, , \\ 0 & \text{if} \quad \alpha = S_\infty \, . \end{cases}$$



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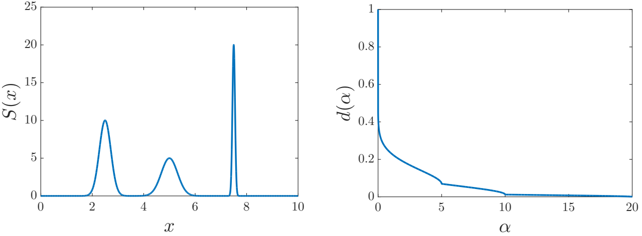
d **is useful** !

It provides a complete description of local maximum sorted from the smallest to the largest.

How to overcome such a "major" drawback in h-method ?

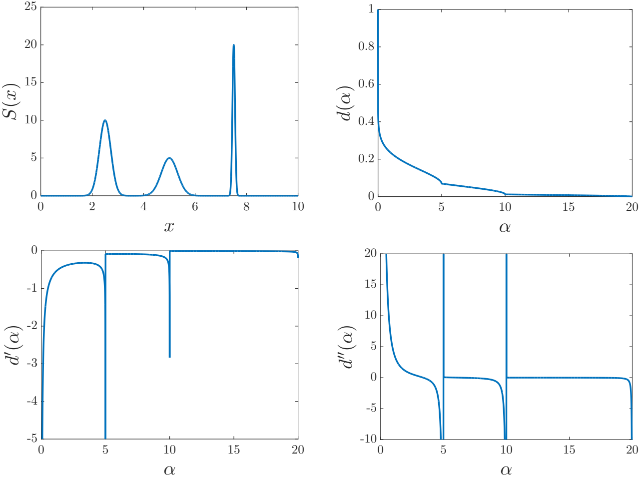
- How to set α ? From d ?



Difficult to choose the threshold from d only

How to overcome such a "major" drawback in h-method ?

- How to set α ? From d ? d' ? or d" ?



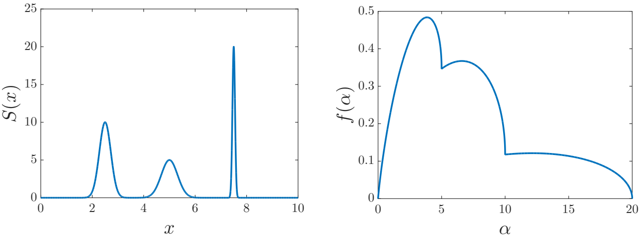
Accurate interpolation is required !

How to overcome such a "major" drawback in h-method ?

- How to set α ? From d ?
- A possible choice : a weighted function $f(\alpha) = \alpha d(\alpha)$

$$\text{Set } \alpha = \alpha_{PE} = \max_{0<\alpha\leq S_m} f(\alpha)$$

- No use of derivatives
- Easy to compute



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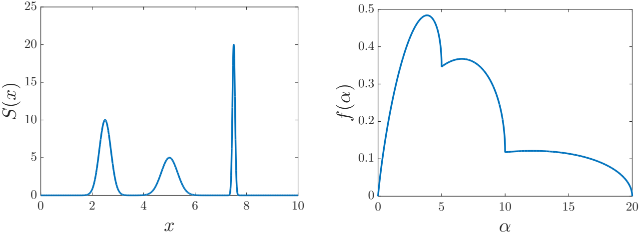
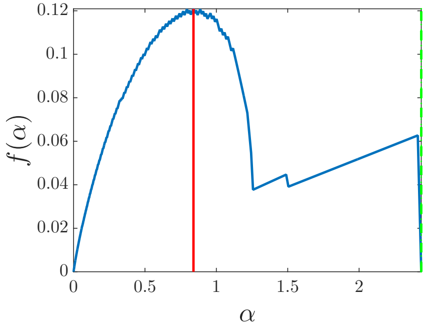
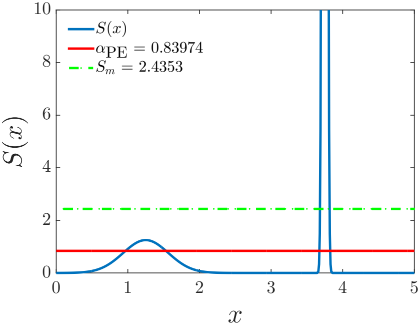
- No use of derivatives
- Easy to compute
- Why the bound S_m ?
  - "Smooth flow/indicator" : $\alpha = S_m$ is generally a good candidate
  - ~~"Discontinuous flow/indicator"~~





(a) $f(\alpha)$



(b) $S(x)$

Figure – The function f for the mesh refinement criterion $S(x) = 200\exp(-1000(x-1.25)^2) + 1.25\exp(-5(x-3.75)^2)$ representing a shock type solution

Illustration : α_{PE} threshold for "smooth" flows

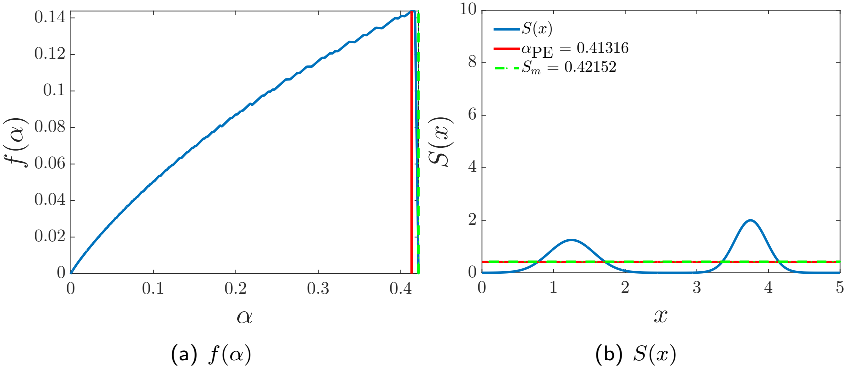
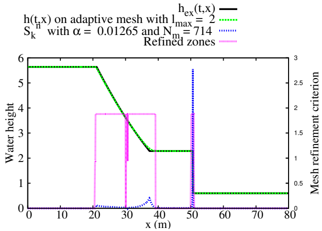


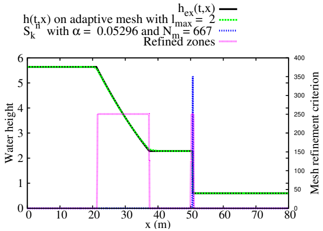
Figure – The function f for the mesh refinement criterion $S(x) = 2\exp(-10(x-1.25)^2) + 1.25\exp(-5(x-3.75)^2)$ representing a smooth flow

Numerical validation : Dam-break problem (Saint-Venant eqs.)

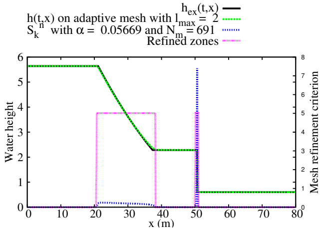
Numerical results for the water height at time $t = 2$ s



(a) $|h_i^n - h_{ex}(x_i, t_n)|$



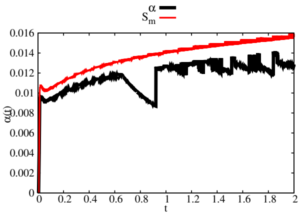
(b) Numerical density of entropy production



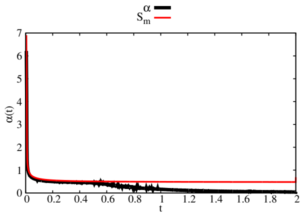
(c) $\left| \dfrac{h_{i+1}^n - h_i^n}{x_{i+1} - x_i} \right|$

Numerical validation : Dam-break problem (Saint-Venant eqs.)

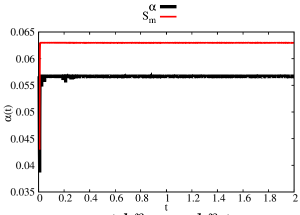
Time evolution of the threshold parameter and the mean value S_m



(d) $|h_i^n - h_{ex}(x_i, t_n)|$



(e) Numerical density of entropy production

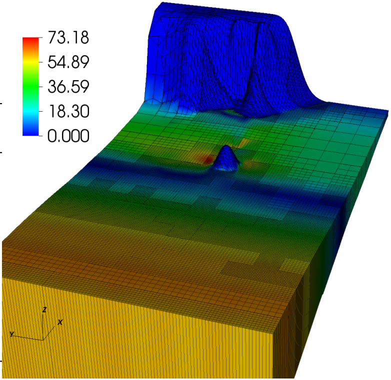


(f) $\left|\dfrac{h_{i+1}^n - h_i^n}{x_{i+1} - x_i}\right|$

Test case : Tsunami runup onto a complex three dimensional Monai Valley

	Adap. sim.	Unif. sim.
T_f	30 s	30 s
Nb. blocks	240	240
Nb. cells	8 000-40 000	62 000
Re-mesh. δt	0.25 s	X
CFL	0.5	0.5

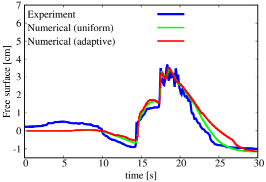
Table – Numerical parameters



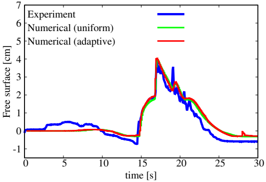
Numerical water height (coloration is issue from the kinetic energy) at $t = 11.25$ s

K. Pons, **M. Ersoy** , F. Golay and R. Marcer.
Adaptive mesh refinement method. Part 2 : Application to tsunamis propagation.

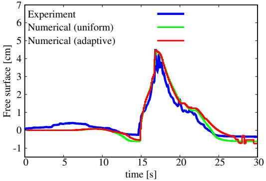
Test case : Tsunami runup onto a complex three dimensional Monai Valley



(g) Gauge 1



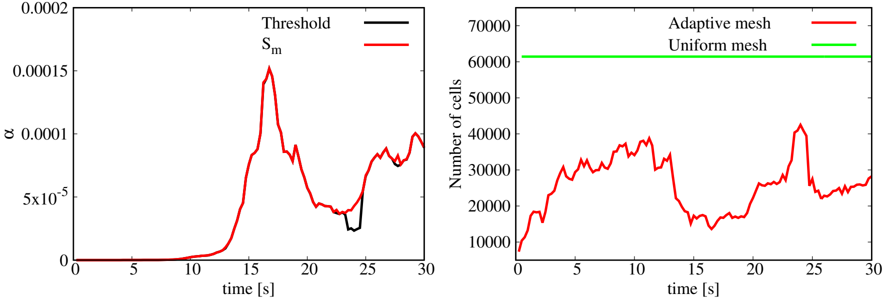
(h) Gauge 2



(i) Gauge 3

Figure – Free surface results at different positions : experimental data versus numerical simulation with and without mesh adaptivity

Test case : Tsunami runup onto a complex three dimensional Monai Valley



(a) Threshold (b) Number of cells

Figure – Time evolution of the mesh refinement threshold and the number of cells : speed up the computation by 3 time

Coming back to the modelling problem : "SVE for certain tsunamis"

- Are the SVE are pertinent for all Tsunamis ?

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  - Land-slide/subaerial landslide generated tsunamis (depending on landslide thickness, water depth) cannot be represented by hydrostatic models ! [a]
  - dispersions are expected

Parisot and Ersoy's experimental wave generator
(Malaga, NumHyp 2019)

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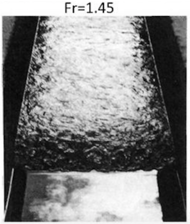

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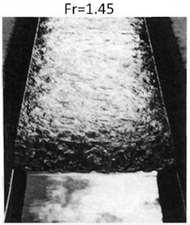

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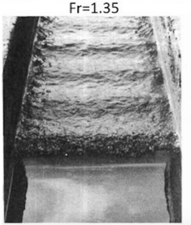

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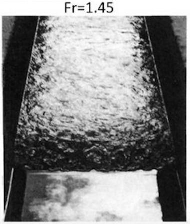

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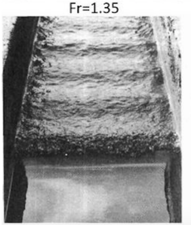

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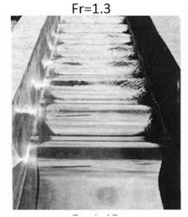

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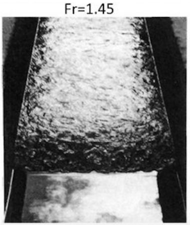

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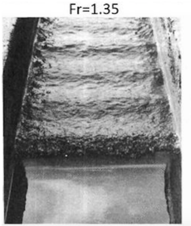

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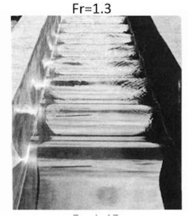

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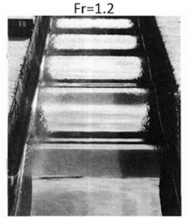

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- Of course, Navier-Stokes equation can deal for both but too costly !

Outline

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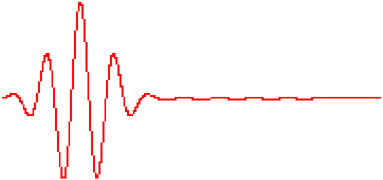
Let $\omega = \frac{2\pi}{T}$ be the angular frequency (pulsation) and $k = \frac{2\pi}{\lambda}$ wavenumber.

- A wave $\phi(kx - \omega t)$ is characterised by two different characteristic speeds
  - phase velocity $C_p = \frac{\omega}{k}$ which corresponds to the displacement of the wave fronts
  - group velocity $C_g = \frac{\partial \omega}{\partial k}$ which corresponds to the displacement of the wave's envelope
  - dispersion relation is given by $\omega = C_p k$
- If C_p is constant then the wave is not dispersive.



Dispersive wave

Non dispersive wave

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- If C_p is constant then the wave is not dispersive.
- According to linear Stokes' theory, noting H the depth, the dispersion relation is

$$\underline{\omega^2 = gk\tanh(kH)} \text{ or } \underline{\lambda = g\frac{T^2}{2\pi}\tanh\left(\frac{2\pi H}{\lambda}\right)} \text{in terms of wavelength } (L =)\lambda$$

Formally, $\dfrac{H}{\lambda} \ll 1$,

- at order 1, $\left(\dfrac{\lambda}{T}\right)^2 = \left(\dfrac{\omega}{k}\right)^2 \approx gH \rightsquigarrow$ SVE

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- at order > 1, $\left(\dfrac{\omega}{k}\right)^2 \approx gH - gk^2H^3 + \ldots \leadsto$ Dispersive models

- Everything starts with Russell's "Wave of translation"

"I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped - not so the mass of water in the channel which it had put in motion ; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height. Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel. Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation". **John** Scott Russell

- Everything starts with Russell's "Wave of translation"
- Proof of the stability of the solitary wave given by Boussinesq (1872)/Korteweg and Gustav de Vries (1895) through a 1D scalar equation : a perfect equilibrium between non-linearities and the dispersive terms

$$u_t + 6uu_x + u_{xxx} = 0$$

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  - **1976** : Green and Naghdi derived the famous 2D fully nonlinear dispersive equations for uneven bottom (1D below)

$$\left\{\begin{array}{l} \dfrac{\partial}{\partial t}h + \dfrac{\partial}{\partial x}(hu) = 0 \\ \dfrac{\partial}{\partial t}(hu) + \dfrac{\partial}{\partial x}\left(hu^2 + \dfrac{h^2}{2F_r^2}\right) + \mu\dfrac{\partial}{\partial x}\left(\dfrac{h^3}{3}\mathcal{D}(u)\right) = 0 \end{array}\right. \quad \text{with}$$

$$\mathcal{D}(u) = \left(\frac{\partial}{\partial x}u\right)^2 - \frac{\partial}{\partial t}\frac{\partial}{\partial x}u - u\frac{\partial}{\partial x}\frac{\partial}{\partial x}u$$

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  - 1976 : Green and Naghdi derived the famous 2D fully nonlinear dispersive equations for uneven bottom.
  - Nowadays : Lannes, Bonneton, Cienfuegos, Dutykh, Gavrilyuk, Richard, Sainte-Marie, . . . proposed several improvements

Coming back to Tsunami propagation : toward a new non-hydrostatic model

- SGN based models are certainly the most appropriate ones for dispersive waves. [a]

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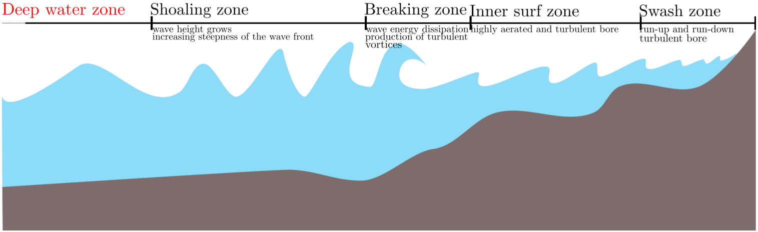
Coming back to Tsunami propagation : toward a new non-hydrostatic model

- SGN based models are certainly the most appropriate ones for dispersive waves.
- But, dispersive and non dispersive waves can coexist during the Tsunami's life . . .
  - Deep water zone : Depth-averaged models hydrostatic and non-hydrostatic models are valid but dispersive codes boosts the CPU times and memory requirements



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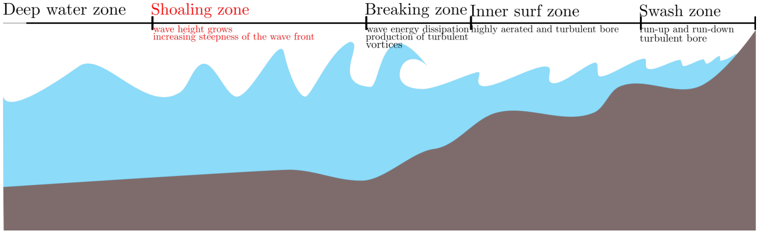
  - Shoaling zone : hydrostatic models are (often) not valid in this zone, leading to an incorrect growth of the wave, yielding to an incorrect prediction of the location of wave breaking



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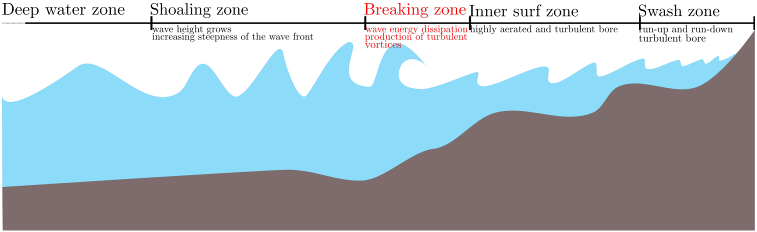
  - Breaking zone : hydrostatic models (SVE) can accurately reproduce broken wave dissipation and swash oscillations without any ad-hoc parametrisation



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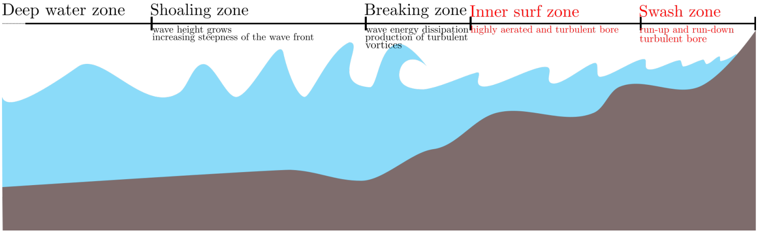
  - Inner surf and swash zones : predominant non-linearities (SVE)

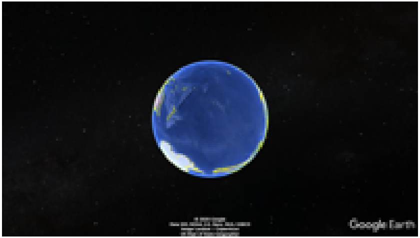


Coming back to Tsunami propagation : toward a new non-hydrostatic model

- SGN based models are certainly the most appropriate ones for dispersive waves.
- But, dispersive and non dispersive waves can coexist during the Tsunami's life ...
- Dissipative models are required [a] : "switching from one model to an other"

a. Lannes, Bonneton, Cienfuegos, Dutykh, Gavrilyuk, Pons, ...

- Waves may penetrate through rivers/channel much faster inland than the coastal inundation reaches over the ground, and may lead flooding in low-lying areas located several km away from the coastline !



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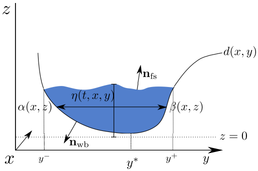
  - 2D models for rivers/channels can be used but costly in the large scale simulation
  - Hydrostatic 1D section-averaged models are well-mastered
  - Non-hydrostatic 1D section-averaged have not yet been derived
    $\rightarrow$ toward the first full non-linear and weakly dispersive section-averaged model

Outline

1. Hydrostatic models, applications and limits
   - Hydrostatic models
   - Application to tsunamis propagation
2. Non-hydrostatic models and applications
   - Historical background and motivations
   - Toward the first dispersive section-averaged model
3. Concluding remarks and perspectives

Incompressible and irrotational Euler equations

$$
\begin{array}{rcl}
\mathrm{div}(\rho_0 \boldsymbol{u}) & = & 0, \\
\dfrac{\partial}{\partial t}(\rho_0 \boldsymbol{u}) + \mathrm{div}(\rho_0 \boldsymbol{u} \otimes \boldsymbol{u}) + \nabla p - \rho_0 \boldsymbol{F} & = & 0
\end{array}
$$



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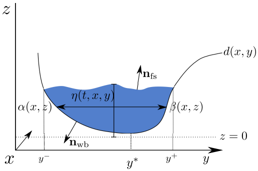
with

$\boldsymbol{u} = (u, v, w)$	:	velocity field
ρ_0	:	density
$\boldsymbol{F} = (0, 0, -g)$	:	external force
p	:	pressure



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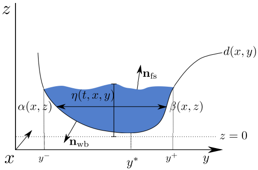

completed with the irrotational relations

$$
\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial z} = \frac{\partial w}{\partial y}, \ \frac{\partial u}{\partial z} = \frac{\partial w}{\partial x} \ .
$$

Incompressible and irrotational Euler equations

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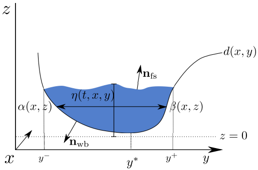


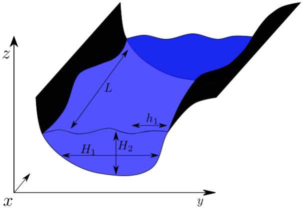
- free surface kinematic boundary condition,

$$\boldsymbol{u}\cdot\boldsymbol{n}_{\mathrm{fs}} = \frac{\partial}{\partial t}\boldsymbol{m}\cdot\boldsymbol{n}_{\mathrm{fs}} \text{ and } p = p_0,\ \forall \boldsymbol{m}(t,x,y) = (x,y,\eta(t,x,y)) \in \Gamma_{\mathrm{fs}}(t,x)$$

- no-penetration condition on the wet boundary

$$\boldsymbol{u}\cdot\boldsymbol{n}_{\mathrm{wb}} = 0,\ \forall \boldsymbol{m}(x,y) = (x,y,d(x,y)) \in \Gamma_{\mathrm{wb}}(x)$$

Let us define the dispersive parameters



- $\mu_1 = \dfrac{h_1^2}{L^2}$
- $\mu_2 = \dfrac{H_2^2}{L^2}$,

such that

$$h_1 < H_1 = H_2 \ll L, \text{i.e. } \mu_1 < \mu_2^2$$

where

H_1	:	characteristic scale of channel width
h_1	:	characteristic wave-length in the transversal direction
H_2	:	characteristic water depth
$F_r = \dfrac{U}{\sqrt{gH_2}}$	:	Froude's number
$T = \dfrac{L}{U}$	:	characteristic time
$\mathcal{P} = U^2$	:	characteristic pressure
X	:	characteristic length of x

Then, define the dimensionless variables

$$\widetilde{x} = \frac{x}{L}, \quad \widetilde{P} = \frac{P}{\mathcal{P}}, \quad \widetilde{\varphi} = \frac{\varphi}{h_1},$$

$$\widetilde{y} = \frac{y}{h_1}, \quad \widetilde{u} = \frac{u}{U}, \quad \widetilde{d} = \frac{d}{H_2},$$

$$\widetilde{z} = \frac{z}{H_2}, \quad \widetilde{v} = \frac{v}{V} = \frac{v}{\sqrt{\mu_1}U}, \quad \widetilde{\eta} = \frac{\eta}{H_2} \ .$$

$$\widetilde{t} = \frac{t}{T}, \quad \widetilde{w} = \frac{w}{W} = \frac{w}{\sqrt{\mu_2}U} \ .$$

We get

$$
\begin{aligned}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= 0 \\
\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial P}{\partial x} &= 0 \\
\mu_1 \left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \right) + \frac{\partial P}{\partial y} &= 0 \\
\mu_2 \left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \right) + \frac{\partial P}{\partial z} &= -\frac{1}{{F_r}^2}
\end{aligned}
$$

and

$$
\frac{\partial u}{\partial y} = \mu_1 \frac{\partial v}{\partial x}, \ \ \mu_1 \frac{\partial v}{\partial z} = \mu_2 \frac{\partial w}{\partial y}, \ \ \frac{\partial u}{\partial z} = \mu_2 \frac{\partial w}{\partial x} \ .
$$

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  $\Rightarrow w(t,x,z) = -\left(\int_d^z u_{|z=d}(t,x)\,dz\right)_x + O(\mu)$
- $\Rightarrow u(t,x,z) = f_1(\bar{u}(t,x)) + \mu f_2(z, \bar{u}(t,x), d(x)) + O(\mu^2)$ where $\bar{u}(t,x) = f_3(u_{|z=d}) \dots$

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Therefore, we assume $\mu_1 < \mu_2$.

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$$\int_{\partial\Omega(t,x)} \left(\frac{\partial}{\partial t} \boldsymbol{M} + u \frac{\partial}{\partial x} \boldsymbol{M} - \boldsymbol{v} \right) \cdot \boldsymbol{n} \; ds = 0$$

Remark II : order of integration

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  - Introduce wet region indicator function Φ which satisfies

$$\frac{\partial}{\partial t}\Phi + \frac{\partial}{\partial x}(\Phi u) + \mathsf{div}_{y,z}\left[\Phi\boldsymbol{v}\right] = 0 \text{ on } \Omega(t) = \bigcup_{0\leq x\leq 1} \Omega(t,x)\ .$$

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    where $\boldsymbol{v} = (v, w)$.
  - Section-average equations using the approximation

$$\begin{array}{lcl} u(t,x,y,z) & = & \bar{u}(t,x) + \mu_2 B_0(\bar{u},x,z) + O(\mu_2^2) \\ \eta(t,x,y) & = & \bar{\eta}(t,x) + O(\mu_1) \\ P(t,x,y,z) & = & P_{\mathrm{h}}(t,x,z) + \mu_2 P_{\mathrm{nh}}(t,x,z) + O(\mu_2^2) \end{array}$$

The new model : generalization of the SGN and Free surface flows equations

$$
\begin{cases}
\frac{\partial}{\partial t}A + \frac{\partial}{\partial x}Q = 0 \\
\frac{\partial}{\partial t}Q + \frac{\partial}{\partial x}\left(\frac{Q^2}{A} + I_1(x,A)\right) + \mu_2\frac{\partial}{\partial x}(\mathcal{D}(u)G(A,x)) = I_2(x,A) \\
+\mu_2\mathcal{G}(u,S,\sigma) + O(\mu_2^2)
\end{cases}
$$

where

$A = \int_{\Omega(t,x)} dy\, dz$	:	wet area
$Q = A(t,x)u(t,x)$	:	discharge
$I_1 = \int_{\Omega(t,x)} \frac{\eta(t,x)-z}{F_r^2}\sigma(x,z)\, dy\, dz$	:	hydro. press.
$I_2 = -\int_{y^-(t,x)}^{y^+(t,x)} \frac{h(t,x)}{{F_r}^2}\frac{\partial}{\partial x}d(x,y)\, dy$	:	hydro. press. source

$$\left\{\begin{array}{l}\frac{\partial}{\partial t}A+\frac{\partial}{\partial x}Q=0\\ \frac{\partial}{\partial t}Q+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}+I_1(x,A)\right)+\mu_2\frac{\partial}{\partial x}(\mathcal{D}(u)G(A,x))=I_2(x,A)\\ +\mu_2\mathcal{G}(u,S,\sigma)+O(\mu_2^2)\end{array}\right.$$

where

$$\mathcal{D}(u)=\left(\frac{\partial}{\partial x}u\right)^2-\frac{\partial}{\partial t}\frac{\partial}{\partial x}u-u\frac{\partial}{\partial x}\frac{\partial}{\partial x}u$$

and

$$G(A,x)=\int_{d^*(x)}^{\eta}\sigma(x,z)\int_z^{\eta}\frac{S(x,s)}{\sigma(x,s)}\,ds\,dz$$

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$$\left\{\begin{array}{l}\dfrac{\partial}{\partial t}A+\dfrac{\partial}{\partial x}Q=0\\ \dfrac{\partial}{\partial t}Q+\dfrac{\partial}{\partial x}\left(\dfrac{Q^2}{A}+I_1(x,A)\right)+\mu_2\dfrac{\partial}{\partial x}(\mathcal{D}(u)G(A,x))=I_2(x,A)\\ +\mu_2\mathcal{G}(u,S,\sigma)+O(\mu_2^2)\end{array}\right.$$

where

$$\begin{aligned}\mathcal{G}(u,S,\sigma) &= \int_z^\eta \frac{u^2}{\sigma(x,s)}\left(\frac{\dfrac{\partial}{\partial x}S(x,s)\dfrac{\partial}{\partial x}\sigma(x,s)}{\sigma(x,s)}-\frac{\partial}{\partial x}\frac{\partial}{\partial x}S(x,s)\right)\\ &\quad+\frac{\partial}{\partial x}\left(\frac{u^2}{2}\right)\frac{S(x,s)\dfrac{\partial}{\partial x}\sigma(x,s)}{\sigma(x,s)^2}\\ &\quad-\left(\frac{\partial}{\partial t}u+u\frac{\partial}{\partial x}u\right)\frac{\dfrac{\partial}{\partial x}S(x,s)}{\sigma(x,s)}\,ds\end{aligned}.$$

$$\left\{\begin{array}{l}\dfrac{\partial}{\partial t}A+\dfrac{\partial}{\partial x}Q=0\\ \dfrac{\partial}{\partial t}Q+\dfrac{\partial}{\partial x}\left(\dfrac{Q^2}{A}+I_1(x,A)\right)+\mu_2\dfrac{\partial}{\partial x}(\mathcal{D}(u)G(A,x))=I_2(x,A)\\ +\mu_2\mathcal{G}(u,S,\sigma)+O(\mu_2^2)\end{array}\right.$$

Setting $\sigma=1$, $d=1$,

- $A=h$
- $S(x,z)\equiv S(z)\Rightarrow\mathcal{G}=0$ and $I_2=0$
- $G=\dfrac{h^3}{3}$
- $I_1=\dfrac{h^2}{2F_r^2}$

$$\left\{\begin{array}{l}\dfrac{\partial}{\partial t}A+\dfrac{\partial}{\partial x}Q=0\\\dfrac{\partial}{\partial t}Q+\dfrac{\partial}{\partial x}\left(\dfrac{Q^2}{A}+I_1(x,A)\right)+\mu_2\dfrac{\partial}{\partial x}(\mathcal{D}(u)G(A,x))=I_2(x,A)\\+\mu_2\mathcal{G}(u,S,\sigma)+O(\mu_2^2)\end{array}\right.$$

we recover the classical SGN equations on flat bottom

$$\left\{\begin{array}{l}\dfrac{\partial}{\partial t}h+\dfrac{\partial}{\partial x}(hu)=0\\\dfrac{\partial}{\partial t}(hu)+\dfrac{\partial}{\partial x}\left(hu^2+\dfrac{h^2}{2F_r^2}\right)+\mu_2\dfrac{\partial}{\partial x}\left(\dfrac{h^3}{3}\mathcal{D}(u)\right)=O(\mu_2^2)\end{array}\right.$$

where

$$\mathcal{D}(u)=\left(\frac{\partial}{\partial x}u\right)^2-\frac{\partial}{\partial t}\frac{\partial}{\partial x}u-u\frac{\partial}{\partial x}\frac{\partial}{\partial x}u$$

The new model : generalization of the SGN and Free surface flows equations

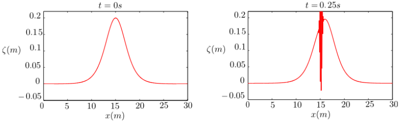
$$\begin{cases} \dfrac{\partial}{\partial t} A + \dfrac{\partial}{\partial x} Q = 0 \\ \dfrac{\partial}{\partial t} Q + \dfrac{\partial}{\partial x}\left(\dfrac{Q^2}{A} + I_1(x,A)\right) + \mu_2 \dfrac{\partial}{\partial x}(\mathcal{D}(u) G(A,x)) = I_2(x,A) \\ +\mu_2 \mathcal{G}(u,S,\sigma) + O(\mu_2^2) \end{cases}$$

Remark

Dispersive equation are usually characterised by third order term

$\Downarrow$

time step restriction and may create high frequencies instabilities



Bourdarias, Gerbi, and Ralph Lteif. Computers & Fluids, 156 :283–304, 2017.

- Define the linear $\mathcal{T}$ and the quadratic $\mathcal{Q}$ operators

$$\mathcal{T}[A,d,\sigma,z](u) = \frac{\partial}{\partial x}(u)\int_z^\eta \frac{S(x,s)}{\sigma(x,s)}\, ds + u\int_z^\eta \frac{1}{\sigma(x,s)}\frac{\partial}{\partial x}S(x,s)\, ds\ ,$$

and

$$\begin{aligned}\mathcal{G}[A,d,\sigma,z](u) \quad = \quad & \int_z^\eta 2\left(\frac{\partial}{\partial x}u\right)^2 \frac{S(x,s)}{\sigma(x,s)} + \\ & \frac{u^2}{\sigma(x,s)}\left(\frac{\frac{\partial}{\partial x}S(x,s)\frac{\partial}{\partial x}\sigma(x,s)}{\sigma(x,s)} - \frac{\partial}{\partial x}\frac{\partial}{\partial x}S(x,s)\right) \\ & + \frac{\partial}{\partial x}\left(\frac{u^2}{2}\right)\frac{S(x,s)\frac{\partial}{\partial x}\sigma(x,s)}{\sigma(x,s)^2}\, ds\end{aligned}$$

- Define the linear $\mathcal{T}$ and the quadratic $\mathcal{Q}$ operators
- Define the averaged linear $\overline{\mathcal{T}}$ and the quadratic $\overline{\mathcal{Q}}$ operators

$$\overline{\mathcal{T}}[A, d, \sigma](u, \psi) = \int_{d^*(x)}^{\eta} \psi \mathcal{T}[A, d, \sigma, z](u) \, dz$$

and

$$\overline{\mathcal{G}}[A, d, \sigma](u, \psi) = \int_{d^*(x)}^{\eta} \psi \mathcal{G}[A, d, \sigma, z](u) \, dz$$

- Define the linear $\mathcal{T}$ and the quadratic $\mathcal{Q}$ operators
- Define the averaged linear $\overline{\mathcal{T}}$ and the quadratic $\overline{\mathcal{Q}}$ operators
- Define the operators $\mathcal{L}$ and $\mathcal{Q}$

$$\mathbb{L}[A,d,\sigma](u) \quad = \quad A\mathcal{L}[A,d,\sigma]\left(\frac{u}{A}\right)$$

and

$$\mathcal{Q}[A,d,\sigma](u) \quad = \quad \frac{1}{A}\left[\frac{\partial}{\partial x}\left(\overline{\mathcal{G}}[A,d,\sigma]\,(u,\sigma)\right) - \overline{\mathcal{G}}[A,d,\sigma]\left(u, \frac{\partial}{\partial x}\sigma\right)\right]$$

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- Reformulated model

$$\left\{ \begin{array}{l} \dfrac{\partial}{\partial t}A + \dfrac{\partial}{\partial x}(Au) = 0 \\ \left(I_d - \mu_2 \mathbb{L}[A,d,\sigma]\right)\left(\dfrac{\partial}{\partial t}(Au) + \dfrac{\partial}{\partial x}\left(Au^2\right)\right) + \dfrac{\partial}{\partial x} I_1(x,A) \\ +\mu_2 A\mathcal{Q}[A,d,\sigma](u) = I_2(x,A) + O(\mu_2^2) \end{array} \right.$$

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REMARK

Inverting $I_d - \mu_2\mathbb{L}[A,d,\sigma]$ ⇒ no third order term ⇒ more stable formulation

- Bonneton, Barthélemy, Chazel, Cienfuegos, Lannes, Marche, and Tissier. European Journal of Mechanics-B/Fluids, 2011
- Debyaoui, Ersoy. Part 2, preprint, 2020

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- and finally the operator $\mathbb{L}$
- Reformulated model

$$\left\{\begin{array}{l} \dfrac{\partial}{\partial t}A + \dfrac{\partial}{\partial x}(Au) = 0 \\ (I_d - \mu_2\kappa\mathbb{L}[A,d,\sigma])\left(\dfrac{\partial}{\partial t}(Au) + \dfrac{\partial}{\partial x}(Au^2) + \dfrac{\kappa-1}{\kappa}\left(\dfrac{\partial}{\partial x}I_1 - I_2\right)\right) \\ +\dfrac{1}{\kappa}\left(\dfrac{\partial}{\partial x}I_1 - I_2\right) + \mu_2 A\mathcal{Q}[A,d,\sigma](u) = O(\mu_2^2) \end{array}\right.$$

REMARK

A consistent one-parameter $\kappa > 0$ family (up to order $O(\mu_2^2)$) can be introduced to improve the frequency dispersion.

- Bonneton, Barthélemy, Chazel, Cienfuegos, Lannes, Marche, and Tissier. European Journal of Mechanics-B/Fluids, 2011
- Debyaoui, Ersoy. Part 2, preprint, 2020

Theorem

Let α,β and $d \in C_b^\infty$ and $A \in W^{1,\infty}(\mathbb{R})$ such that $\inf_{x\in\mathbb{R}} A \geq A_0 > 0$. Then the operator

$$\mathbb{T} : H^2(\mathbb{R}) \to L^2(\mathbb{R})$$

is well-defined, one-to-one and onto.

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- Let $\mu_2 \in (0, 1)$. Define the space $H^1_{\mu_2}(\mathbb{R})$ the space $H^1(\mathbb{R})$ endowed with the norm

$$\| u \|_{\mu_2}^2 = \| u \|_2^2 + \mu_2 \| u_x \|_2^2$$

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- Let $\mu_2 \in (0,1)$. Define the space $H^1_{\mu_2}(\mathbb{R})$
- Define the bilinear form $a(u,v)$

$$a(u,v) = (A\mathbb{T}u, v) = (Au, v) +$$

$$\mu_2 \left(A \left(\frac{A}{\sqrt{3}u_x} - \frac{\sqrt{3}}{2} d_x u \right), \left(\frac{A}{\sqrt{3}v_x} - \frac{\sqrt{3}}{2} d_x v \right) \right) + (A d_x u, d_x v)$$

Invertibility of the operator $\mathbb{T} = A(I_d - \mu_2 \mathbb{L}[A, d, \sigma])$

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is well-defined, one-to-one and onto.

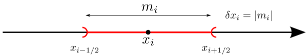
- Let $\mu_2 \in (0, 1)$. Define the space $H^1_{\mu_2}(\mathbb{R})$
- Define the bilinear form $a(u, v)$
- Lax-Milgram theorem

$$\exists!\ u \in H^1_{\mu_2}(\mathbb{R})\ ;\ a(u, v) = (f, v),\ \forall v \in H^1_{\mu_2}(\mathbb{R}),\ f \in L^2(\mathbb{R})$$

$$\Downarrow$$

$$\exists!\ u \in H^1_{\mu_2}(\mathbb{R})\ ;\ \mathbb{T}u = f$$

- From definition of $\mathbb{T}$, we get $u_{xx} = g(A, u, d, \sigma) \in L^2(\mathbb{R}) \Rightarrow u \in H^2(\mathbb{R})$.



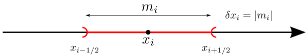
We consider a classical Finite Volume scheme, $\boldsymbol{U} = (A, Q)$

$$\boldsymbol{U}_i^{n+1} = \boldsymbol{U}_i^n - \frac{\delta t^n}{\delta x}\left(\boldsymbol{F}_{i+1/2}(\boldsymbol{U}_i^n, \boldsymbol{U}_{i+1}^n) - \boldsymbol{F}_{i-1/2}(\boldsymbol{U}_{i-1}^n, \boldsymbol{U}_i^n)\right)$$

where $\boldsymbol{F}_{i\pm1/2} \approx \dfrac{1}{\delta t^n}\displaystyle\int_{m_i} \boldsymbol{F}(\boldsymbol{U}(t, x_{i+1/2}))\, dx$ is a Finite volume solver,

with

$$\boldsymbol{F}(\boldsymbol{U}) = \begin{pmatrix} Au \\ Au^2 + \dfrac{\kappa - 1}{\kappa}\left(I_1 -'' \displaystyle\int I_2''\right) \end{pmatrix}$$



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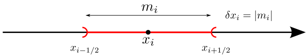
where $\boldsymbol{F}_{i\pm1/2} \approx \dfrac{1}{\delta t^n}\displaystyle\int_{m_i} \boldsymbol{F}(\boldsymbol{U}(t, x_{i+1/2}))\, dx$ is a Finite volume solver, for instance, with upwind technique to deal with source term

$$\boldsymbol{F}_{i\pm1/2} = \frac{\boldsymbol{F}(\boldsymbol{U}) + \boldsymbol{F}(\boldsymbol{V})}{2} - \frac{s_i^n}{2}(\boldsymbol{V} - \boldsymbol{U})$$

with

$$\boldsymbol{F}(\boldsymbol{U}) = \begin{pmatrix} Au \\ Au^2 + \dfrac{\kappa - 1}{\kappa}\left(I_1 -'' \int I_2 ''\right) \end{pmatrix}$$

- Bourdarias, Ersoy, Gerbi. Journal of Scientific Computing, 2011



We consider a classical Finite Volume scheme, $\boldsymbol{U} = (A, Q)$

$$\boldsymbol{U}_i^{n+1} = \boldsymbol{U}_i^n - \frac{\delta t^n}{\delta x}\left(\boldsymbol{F}_{i+1/2}(\boldsymbol{U}_i^n, \boldsymbol{U}_{i+1}^n) - \boldsymbol{F}_{i-1/2}(\boldsymbol{U}_{i-1}^n, \boldsymbol{U}_i^n)\right)$$

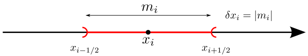
$$-\frac{\delta t^n}{\delta x}\left(\left[(I_d - \mu_2 \mathbb{L})^n\right]^{-1} \boldsymbol{D}^n\right)_i$$

with

$$(\boldsymbol{D}^n)_i = \boldsymbol{D}_{i+1/2}(\boldsymbol{U}_{i-1}^n, \boldsymbol{U}_i^n, \boldsymbol{U}_{i+1}^n) - \boldsymbol{D}_{i-1/2}(\boldsymbol{U}_{i-2}^n, \boldsymbol{U}_{i-1}^n, \boldsymbol{U}_i^n)$$

where $\boldsymbol{D}_{i\pm1/2}$ and $\left[(I_d - \mu_2 \mathbb{L})^n\right]^{-1}$ are the centred approximation of

$$\mathcal{D} = \frac{1}{\kappa}\left(\frac{\partial}{\partial x} I_1 - I_2\right) + \mu_2 A \mathcal{Q} \text{ and } \left[(I_d - \mu_2 \mathbb{L})\right]^{-1}$$



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THEOREM

The numerical scheme is stable under the classical CFL condition,

$$\max_{\lambda \in \mathrm{Sp}(D_{\boldsymbol{U}} \boldsymbol{F}(\boldsymbol{U}))} |\lambda| \frac{\delta t^n}{\delta x} \leqslant 1 \ .$$

- Debyaoui, Ersoy. NumHyp, 2020

- Influence of the Section Variation ($N = 5000$ cells) :
  $\sigma(x;\varepsilon) = \beta(x;\varepsilon) - \alpha(x;\varepsilon)$ with
  $\beta = \dfrac{1}{2} - \dfrac{\varepsilon}{2} \exp\left(-\varepsilon^2 \left(x - L/2\right)^2\right)$ and $\alpha = -\beta$

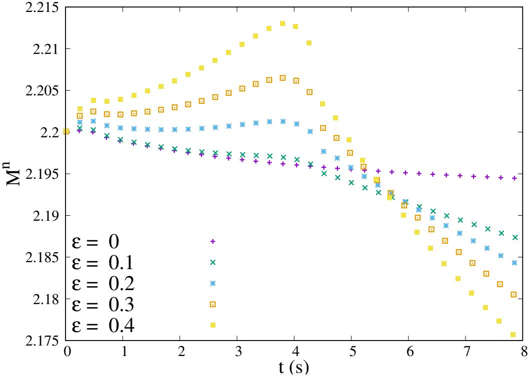

Figure – $M^n := \max_{x\in[0,L_c]} (h_i^n)$

- Influence of the Section Variation ($N = 5000$ cells) :
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  $\beta = \frac{1}{2} - \frac{\varepsilon}{2}\exp\left(-\varepsilon^2\left(x - L/2\right)^2\right)$ and $\alpha = -\beta$
- Numerical order for $\varepsilon = 0$

	$\Vert \eta\text{num} - \eta_{exact} \Vert_2$	$\Vert \eta\text{num} - \eta_{exact} \Vert_\infty$
Order	0.53	0.58

- Numerical order for $\varepsilon = 0.4$ (reference solution obtained with $N = 10000$ cells)

	$\Vert \eta\text{num} - \eta_{\text{ref}} \Vert_2$	$\Vert \eta\text{num} - \eta_{\text{ref}} \Vert_\infty$
Order	0.64	0.56

- Comparison with the NLSW and the exact solution

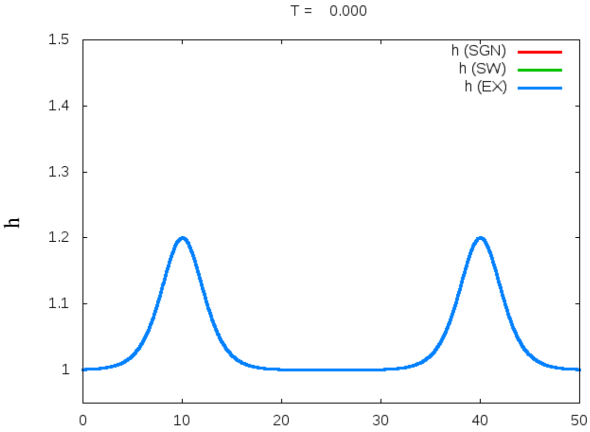
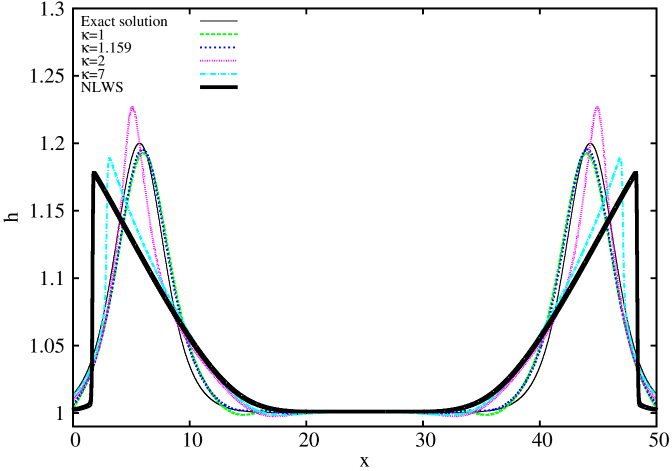


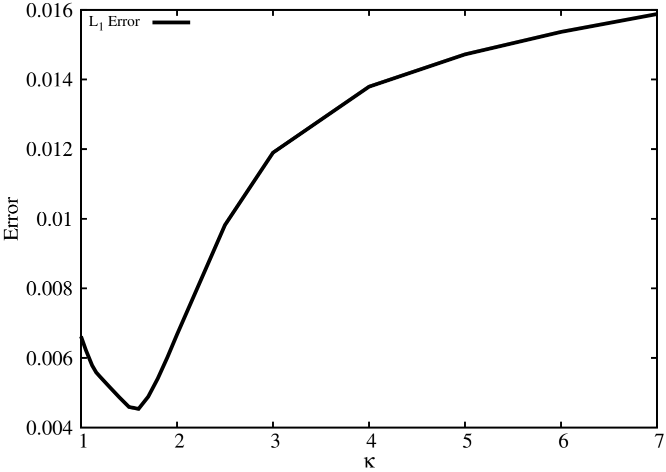
Figure – $\sigma = 1$, $d = 1$, $N = 1000$, $CFL = 0.95$, $T_f = 10$ and $\kappa = 1.159$

- Comparison with the NLSW and the exact solution
- Influence of κ



(b) Solutions at time $T_f = 10$

- Comparison with the NLSW and the exact solution
- Influence of κ



(d) $\| h_{ex} - h_\kappa \|_1$

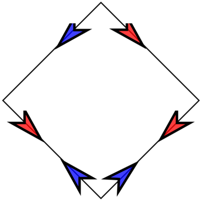
Outline

Outline

- Flood risks, flooding by waves, monitoring the evolution of the coastline, ...

Meteorological model
Rain,wind, pressure, ...
Primitive equations

Hydrological model and hydrogeological
Saturation levels of permeable soils, water table levels, retention of water, runoff, transport of sediment, rivers, ...
Free surface models, Richards equations, Saint-Venant-Exner



Urban hydraulic models
Runoff on heterogenous permeable/impermeable soils, infiltration of underground networks, underground flows, geysers, ...
Transient mixed flows in networks (PFS model)

Marine models
Waves, Tsunamis, Coastline
Free surface model, Saint-Venant, Euler

M. Ersoy, T. Ngom, M. Sy
Compressible primitive equation : formal derivation and stability of weak solutions.
Nonlinearity, 2011

M. Ersoy, T. Ngom,
Existence of a global weak solution to one model of Compressible Primitive Equations.
Comptes Rendus Mathématique, 2012

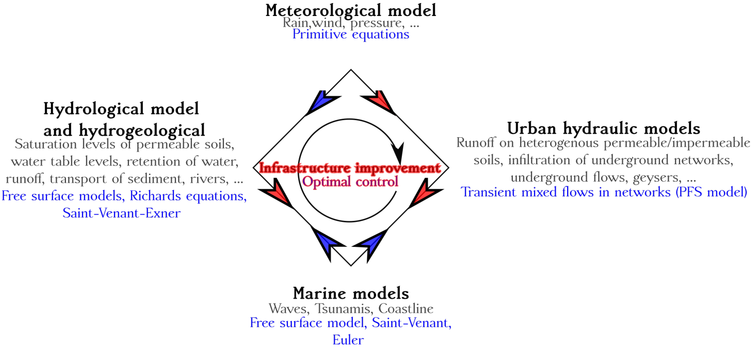
C. Bourdarias, **M. Ersoy**, S. Gerbi
Air entrainment in transient flows in closed water pipes : a two-layer approach.
ESAIM : Mathematical Modelling and Numerical Analysis, 2013

C. Bourdarias, **M. Ersoy**, S. Gerbi
Unsteady mixed flows in non uniform closed water pipes : a Full Kinetic Appraoch.
Numerische Mathematik, 2014

F. Golay, **M. Ersoy**, L. Yushchenko, D. Sous
Block-based adaptive mesh refinement scheme using numerical density of entropy production for three-dimensional two-fluid flows.
International Journal of Computational Fluid Dynamics, 2015

Interconnected modelling problems tackled

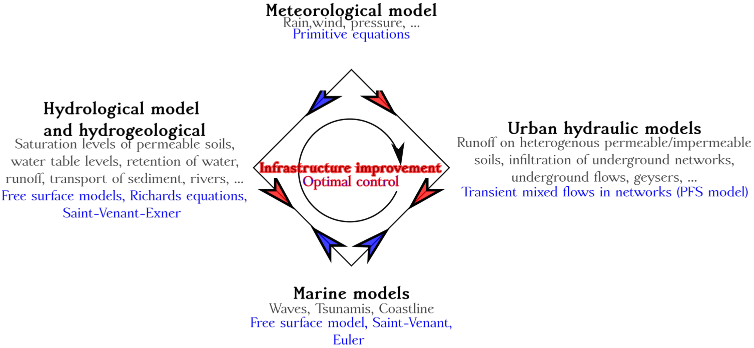
- Flood risks, flooding by waves, monitoring the evolution of the coastline, ...



- Optimal control : for instance,
  - breaking/absorbing wave energy
  - generating friction/filtration to reduce the risk of flooding
  - dimensioning underground networks

M. Ersoy, E. Feireisl, E. Zuazua
Sensitivity analysis of 1-d steady forced scalar conservation laws.
Journal of Differential Equations, 2013

- Flood risks, flooding by waves, monitoring the evolution of the coastline, ...



- Optimal control for infrastructure improvement
- Leading to mathematical (well-posedness, special solutions, stability, control, coupling, ...) and numerical challenges (stability, convergence, well-balanced, high order scheme, drying/flooding, multi-scale code, FV, DG, ...).

Thank you

for your

attention